List of Abstracts for

MINGLE 2003

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Contents

Introduction

The MINGLE 2003 workshop includes all of the papers listed here plus three invited talks by Dr Hugues Hoppe (Microsoft Research), Dr Ulrich Reif (Darmstadt) and Dr Jörg Peters (Florida). Further information about the workshop and details of how to register can be found at the workshop website http://www.mingle2003.org

There are three types of presentation at MINGLE 2003: the three invited talks; seven survey papers, which summarise the recent research in the various aspects of multiresolution modelling; and twenty-three new research papers, which present the latest research in the field.

Compression of 3D Meshes

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3D meshes are widely used in graphic and simulation applications for approximating 3D objects. Representing complex shapes requires a large amount of data for storage and transmission in a raw format. Applications calling for compact storage and fast transmission have motivated the development of many algorithms to efficiently compress 3D meshes. Here we survey the more recent developments in compression of both the connectivity and geometry components of 3D meshes. We mainly compare the main ideas and intuition behind state-of-the-art techniques for single-rate and progressive mesh encoding. For some of the techniques, we discuss theoretical asymptotic behaviour or conjectures of optimality. We also list some theoretical and practical open questions and directions for future research in mesh and shape compression.

Multiresolution editing of curves and surfaces under constraints

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Multiresolution analysis has received considerable attention in recent years in many fields of computer graphics, geometric modelling and visualisation. It provides a powerful tool for efficiently representing functions at multiple levels of detail. General unconstrained multiresolution editing or deformation techniques for parametric curves have been explored in the past. However, there are many application areas, including CAD/CAM and computer animation, where deformations under constraints are needed. Constraint enforcement offers additional control of a shape during the modelling process. Constraining *linear geometric properties*, like position, normal and point tangent and *non-linear constraints* like area, length, volume combined with minimising curve or surface energy prove to be an effective tool both for sculpting models and for animating real behaviours of objects. After reviewing briefly general modelling techniques under constraints the paper will survey recent advances in constrained multiresolution editing and modelling techniques of curves and surfaces.

A survey on data structures for level-of-detail models

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Due to the rapidly increasing complexity of three-dimensional datasets such as geometric free-form shapes, terrain models or volumetric scalar fields, the investigation of hierarchical methods to control and adjust the level of detail (LOD) of a given dataset has been an active research area – and it still is. Several sophisticated and highly efficient data structures have been developed to store and access the geometrical and topological information of LOD representations. In this paper we survey some of the major approaches.

We classify LOD data structures according to the dimensionality of the basic structural element into point-, edge-, triangle-, and tetrahedron-based. Within each class there will be general data structures for irregular models as well as more specialised data structures that assume a certain (semi-) regularity of the data.

Adaptive thinning algorithms

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Thinning algorithms are greedy point removal schemes for scattered data, where the points are recursively removed according to one specific removal criterion. In *adaptive* thinning algorithms, the removal criterion depends on both the locations of the data points and function values, one attached to each point. Therefore, adaptive thinning necessarily works with *functional data*. This is in contrast to *non-adaptive* thinning, where the point removal depends only on the geometry of the given point set.

Adaptive thinning algorithms were originally designed for the purpose of approximating both univariate and bivariate scattered data by linear splines. In either case, this amounts to compute subsets of the points, such that a corresponding linear spline interpolant is close to the original data. In the univariate case, this linear spline interpolant is already well-defined by the selection of the points. In the bivariate case, a convenient choice is the piecewise linear interpolant over the Delaunay triangulation of the selected point set. In both cases, adaptive thinning maintains for every point in the current subset an *anticipated error*, which measures an estimate of the error incurred by its removal. A point which minimises the anticipated error is considered to be *least significant*, and so it is removed. In this way, adaptive thinning constructs a data hierarchy from the input data, which in turn provides a multiresolution representation of the target function by a sequence of linear splines.

Adaptive thinning algorithms are useful for model simplification and data compression. In fact, adaptive thinning has very recently been used for the compression of digital images. In this particular application, adaptive thinning serves to construct a scattered subset of most significant pixel positions from the given image. These pixel positions, along with their attached luminance values, are then converted into a bit-stream by using a customised coding scheme for scattered data. At the decoder, the transmitted data is used for the image reconstruction by evaluating a linear spline over the Delaunay triangulation of the most significant pixels.

This survey first provides a general introduction to adaptive thinning algorithms, before several specific such algorithms are discussed in detail. Both computational and theoretical aspects are addressed. Finally, special emphasis is put on the application of adaptive thinning to digital image compression. The performance of this novel compression scheme is comparable to the well-established wavelet-based compression method SPIHT, and is even better for geometrical images with sharp edges.

Recent progress in subdivision — a survey

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Subdivision surfaces were first described by Catmull and Clark in 1978, soon, in fact, after the now-ubiquitous NURBS were identified as being a sensible standard for parametric surface descriptions. For twenty years they were an interesting generalisation of (a subset of) NURBS, with a paper on one aspect or another appearing in some relevant journal every now and then.

In the last five years the situation has totally changed. Subdivision surfaces are now one of the methods of choice in Computer Graphics, and some consider that they might supplant NURBS as the standard in engineering CAD.

This paper looks at some of the technical changes that have happened in the last five years or so which are helping to drive that change in status.

We divide our paper into the following aspects:

- Background - identifying the "classical" knowledge

- New schemes and a classification of schemes
- New domains - a subdivision surface may be thought of as a map from a bivariate manifold into $R3$. We can map from other domains e.g. from a trivariate manifold.
- New ranges - we can also map into other ranges.
- New issues - the focus on smoothness has now been joined by other criteria for judging the quality of a scheme.
- New ideas - it has become clear that linear stationary schemes may not provide the solution to all our problems, and newer ideas broaden our horizons.
- Conclusions - the areas in which future work might be most fruitful.

Surface parameterization

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A parameterization of a surface can be viewed as a one-to-one mapping from the surface into a suitable parameter domain. Typically, surfaces that are homeomorphic to a disc are mapped into the plane. In general, the parameter domain will itself be a surface, and so parameterization means mapping one surface into another.

Such parameterizations have many applications, among them: parametric scattered data fitting; texture mapping; morphing; remeshing; reparameterization of spline surfaces; and repair of CAD models.

Parameterizations almost always introduce distortion in either angles or areas and a good mapping in applications is one which minimises these distortions in some sense. Many different ways of achieving this have been proposed in the literature. Usually the surfaces are either represented by or approximated by triangular meshes, and the mappings are piecewise linear.

A great many papers have been written on this topic recently, and we will give an overview of the main developments.

Polynomial and spline multiresolution analysis and the lifting scheme

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Over the years the authors have studied together with various collaborators finite-dimensional multiresolution analysis, for example based on polynomial splines [LMQ] and trigonometric [PQ] as well as algebraic polynomials [KP]. In this lecture we would like to investigate how these specific approaches relate to a well-known general framework for the construction of wavelets, namely the lifting scheme [SchSw].

[KP] Kilgore, T., Prestin, J.: Polynomial wavelets on the interval. Constr. Approx. 12, 95–110 (1996)

[LMQ] Lyche, T., Mørken, K., Quak, E.: Theory and algorithms for nonuniform spline wavelets. Multivariate Approximation and Applications, N. Dyn, D. Leviatan, D. Levin and A. Pinkus (eds.), Cambridge University Press, 152–187 (2001)

[PQ] Prestin, J., Quak, E.: Trigonometric interpolation and wavelet decompositions. Num. Alg. 9, 293–317 (1995)

[SchSw] Sweldens, W., Schröder, P.: Building your own wavelets at home. Wavelets in Computer Graphics, ACM SIGGRAPH Course Notes (1996)

Optimising 3D triangulations for improving the initial triangulation for the butterfly subdivision scheme

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This talk is concerned with the construction of a "good" 3D triangulation of a given set of points in 3D, to serve as an initial triangulation for the generation of a well shaped surface by the butterfly scheme. The constructed triangulation is "optimal" in the sense that it locally minimises a cost function. We used two algorithms for obtaining a locally-optimal triangulation by swapping edges, one does it sequentially and the other uses a priority queue, and sweeps edges which correspond to a maximal reduction in the cost function. We tested some cost functions which measure certain discrete curvatures of the surface generated by the butterfly scheme. The discrete curvatures are based on the normals to this surface at the given 3D vertices. These normals can be expressed explicitly in terms of the vertices and the connectivity between them in the initial mesh. It is observed from numerical simulations that these optimisation procedures lead to good results for triangulations of vertices sampled from convex objects. It fails in case of non-convex objects, yet, it improves the convex regions in the generated surfaces. In a new approach, for the case of data consisting of 3D vertices and corresponding normals, the cost function measures the deviations of the normals to the butterfly surface from the given normals. This new approach improves the optimal triangulation for data sampled from non-convex object.

Error-bounded simplification of topologically-complex assemblies

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In this paper we present a new simplification approach intended for scenes containing a huge number of simple objects forming a topologically-complex assembly. Our method combines appearance preservation and topology reduction by converting a 3D model to and from an intermediate octree representation. Unlike related approaches, the inside/outside values at octree vertices are computed according to neighbourhood configuration rather than by direct sampling. This allows the reconstructed surface to span only a reduced subset of the terminal nodes of the octree (those which are classified as border nodes), thus avoiding small cracks and removing internal structures not visible from the outside. The reconstruction step of our method succeeds in preserving the appearance of most of the scene objects while drastically simplifying the geometry and topology.

Simple computation of the eigencomponents of a subdivision matrix in the frequency domain

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The convergence behaviour of a subdivision scheme at an extraordinary vertex (EV) is completely defined by the eigencomponents of its subdivision matrix [DS78]. The analysis and the improvement of a subdivision scheme in the vicinity of such singularities of the control mesh hence require a simple technique for identifying and computing the various eigencomponents. The standard analysis method exploits the scheme's rotational symmetries through the use of the Fourier transform. This partitions the subdivision matrix, whose size varies linearly with the valence of the EV, into a block diagonal matrix. Although the values in the blocks and the number of blocks depend on the valence, the blocks are of a fixed size, and so it becomes possible to determine the eigencomponents of all valencies with a single algebraic computation. For instance, the dominant eigencomponents coming from the frequency block $\omega = 1$ define the tangential behaviour of a scheme around an EV.

In this paper, we present a simple method which allows a fast computation of the different frequency blocks. Our approach is illustrated on Kobbelt's $\sqrt{3}$ [Kob00] scheme and we show how very simple computations performed on a single subdivision iteration rather than on the square of the subdivision matrix allow us to deal with the scheme's rotation property and to find the standard subdivision rules. We then extend the common *one*-ring analysis to a larger *two*-rings neighbourhood in order to perform our analysis on larger subdivision matrices. From this larger configuration, we will summarise and discuss different techniques for modifying and improving the subdivision scheme in the vicinity of an EV.

[DS78] Doo, D., Sabin, M.A., Analysis of the behaviour of recursive subdivision surfaces near extraordinary points. Computer Aided Design, **10** (6), 356–360 (1978)

[Kob00] Kobbelt, L., $\sqrt{3}$-Subdivision. Proceedings of SIGGRAPH 2000, Computer Graphics Proceedings, Annual Conference Series, ACM, 103–112 (2000)

MRA frames on bounded intervals

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Hilbert frames are *overcomplete* and *stable* families of functions from a Hilbert space which provide (not necessarily unique) series representations for each element of the space. Frames play an important role in signal processing and other areas of applied mathematics and they can be considered to be a natural generalisation of Riesz bases. The overcompleteness of the system incorporates redundant information in the frame coefficients, such that the loss of some of them does not necessarily imply loss of information.

A sequence of functions $\{f_i\}_{i \in I\!N}$ from a separable Hilbert space $\mathcal{H}$ (usually chosen as L_2) is called a *Bessel sequence* if there exists a constant B ($0 <$

$B < \infty$) such that for every $f \in \mathcal{H}$

$$\sum_{i=1}^{\infty} | < f, f_i > |^2 \leq B \cdot \|f\|_{\mathcal{H}}^2. \tag{1}$$

A Bessel sequence with Bessel bound B is called a *frame* if, in addition, there exists a constant A ($0 < A \leq B < \infty$) such that for every $f \in \mathcal{H}$ one has

$$A \cdot \|f\|_{\mathcal{H}}^2 \leq \sum_{i=1}^{\infty} | < f, f_i > |^2. \tag{2}$$

A and B are the lower and the upper frame bound, respectively, and the inequalities (1) and (2) together are called the *frame condition* and they express the *stability* of the function family.

If one can choose equal frame bounds then the frame is called *tight*. A tight frame can always be *normalised*, such that $A = B = 1$. The tight frames generalise the orthonormal wavelets.

Two frames $\{f_i\}_{i \in I\!N}$ and $\{g_i\}_{i \in I\!N}$ from the same separable Hilbert space $\mathcal{H}$ are dual to each other if for all $f, g \in \mathcal{H}$ we have the identity

$$< f, g > = \sum_{i=1}^{\infty} < f, f_i >< g_i, g > . \tag{3}$$

Every frame has a dual, but it is not necessarily uniquely defined. The tight frames are a very important subclass of frames, because the canonical dual of a tight frame is very easy to determine, e.g. the dual of a normalised tight frame is the frame itself. In the MRA setting a frame together with a dual, where both f_i and g_i are elements of the *same* “spline” space, are called *sibling frames*.

We present a general construction scheme for wavelet frames and corresponding sibling duals, which are defined from a non-stationary MRA, where both irregular shifts on the same refinement level and nonuniform refinement between levels are allowed. Our construction yields analytical formulations of the frame elements and enables us to choose frame elements with local support and maximal order of vanishing moments, features that are important in applications.

The generic example of this construction is that of tight spline frames on a bounded interval I, where the MRA of $L_2(I)$ is given by nested knot sequences, which are dense in I. The fact that only methods from spline theory (such as knot insertion) and from linear algebra are needed in order to develop the theory (*no Fourier techniques!*), allows *arbitrary* nested knot sequences with multiple knots instead of uniformly refined ones. In the sibling case, the spline frame will have a *spline* dual, a property which cannot be achieved in the orthonormal wavelet case.

[1] C.K. Chui, W. He, J. Stöckler: *Non-stationary tight wavelet frames on bounded intervals*, submitted. Temporary reference: Universität Dortmund, Ergebnisberichte Angewandte Mathematik Nr. 230 (April 2003).

[2] C.K. Chui, J. Stöckler: *Recent development of spline wavelet frames with compact support*, appears in: "Beyond Wavelets" (ed. by G.V. Welland), Academic Press 2002.

A data structure for unstructured multi-resolution tetrahedral meshes

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Several applications, including scientific visualisation, medical imaging, and finite element analysis, deal with increasingly large sets of three-dimensional data describing scalar fields, called volume data sets. In order to analyse volume data sets of large size and to accelerate their rendering, a multi-resolution approach can be used. Multi-resolution meshes have been extensively apply for describing surfaces and two-dimensional height fields. They encode the steps performed by a simplification process within a compact structure, in such a way that a virtually continuous collection of simplified meshes at different Levels-Of-Detail (LODs) can be extracted on-line. By applying a multi-resolution approach to tetrahedral meshes, we may have the resolution (i.e., the density of the cells) of the approximating mesh varying in different parts of the field domain (e.g. inside a box, or along a cutting plane), or in the proximity of interesting field values. This will enable a user to interactively explore large volume data using simplified approximations, and to inspect specific areas of interest.

In our previous work, we have defined a general multi-resolution model based on d-dimensional simplicial meshes, called a Multi-Tessellation (MT), which is both dimension- and application-independent, and provides a framework for continuous multi-resolution mesh-based modelling based on a collection of updates and on a dependency relation. Here, we consider an instance of such a model for multi-resolution tetrahedral meshes built through a common simplification operation, half-edge collapse, and we call such a model a Half-Edge Multi-Tessellation (Half-Edge MT). We discuss a data structure for a Half-Edge MT, and in particular a compact implicit representation for the updates and we compare alternative ways for encoding the dependency relation. We show that this data structure provides very good compression ratios also with respect to encoding the original mesh at full resolution. We also describe algorithms for performing incremental selective refinement on such data structure. Selective refinement is the process of extracting variable-resolution meshes from a multi-resolution model and is the common strategy used for answering queries in LOD modelling.

A comparative study of some current wavelet software

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For anyone considering to write programs or libraries for wavelet-based applications, general-purpose wavelet packages are already available for high-level languages like Matlab, Mathematica and XploRE. There are also several more specialised libraries in low-level languages like C++. Even if the purpose is to write completely new programs or libraries, it can still be worthwhile to first get an overview of already existing software, in order to avoid reinventing the wheel or repeating old mistakes.

The following sections contain selected parts of such a comparative overview. This study was done in the initial phase of the development of a C++ library for wavelet-based geometric modelling. Our main focus is on some general-purpose Matlab toolboxes and on some more specialised C++ libraries.

Multi-scale representation of geographic maps based on plane graphs

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We address the problem of representing geographic maps in vector format and at different scales. Vector maps can be represented effectively through plane graphs. In this context, spatial entities forming a map, together with the topological relations among them, can be encoded efficiently in a topological data structure.

In a computer environment, the concept of scale is related to the level of detail of entities represented in a map for a given area (the larger the scale, the higher the level of detail), rather than to the classical notion of scale. Modern Geographic Information Systems (GISs) deal with maps at different scales, which can ideally span from a global (worldwide) scale to a very local (single house) scale. A GIS should be able to relate representations of the same entities in different maps and to generate dynamically new maps which contain entities at different scales.

In previous work, it has been shown that maps at a large scale represented with plane graphs can be generalised to a smaller scale through a restricted set of operators that modify the plane graph. Such operators support the encoding of relations between representations of the same entity at different scales, by consistent mapping of entities from the input map onto entities in the generalised map.

In this work we develop a framework and a data structure to represent multi-scale maps and we propose an algorithm for extracting maps at variable scale from this framework. We consider an input map at large scale that is generalised through the sequential application of operators until a drastically generalised map is obtained. We record the sequence of generalisation updates and we define a partial order among them. The resulting framework can be represented as a Directed Acyclic Graph (DAG) having updates as its nodes and direct dependency links as its arcs.

The data structure for encoding the multi-scale framework consists of a compact representation of the DAG, plus a compressed scheme for representing the generalisation operators and their reverse operators, which are nodes in the DAG. This data structure supports the incremental construction of a topological data structure for the variable-scale map extracted through the algorithm outlined below.

The algorithm for extracting variable scale maps is based on a top-down traversal of the DAG starting at its root, which corresponds to the most generalised map. For each node traversed, the corresponding local generalisation update is possibly undone (i.e., the map is locally refined), depending on input parameters that specify the desired level of detail.

An algorithm for decomposing multi-dimensional non-manifold objects into nearly manifold components

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We address the problem of building valid representations for non-manifold d-dimensional objects. To this aim, we consider an approach based on decomposing a non-manifold d-dimensional object into an assembly of more regular components. We describe the combinatorial structure of non-manifold objects using through abstract simplicial complexes. We propose a well-sounded decomposition of abstract simplicial complexes, describing d-dimensional non-manifold objects, into components that belong to a well-understood class, that we call initial quasi-manifolds. The class of initial quasi-manifolds forms a decidable superset of d-manifolds for d >= 3 and is the same as d-manifolds for d <= 2. Such a decomposition, that we call the standard decomposition of the complex, can be shown to be unique. We present an algorithm for computing a standard decomposition of an abstract cell complex, and we show that the algorithm has a worst-case time complexity linear in the total number of simplexes. Our approach provides a better understanding of the combinatorial structure of non-manifold objects, as well as a rigorous basis for designing efficient dimension-independent data structures for describing non-manifold objects.

Geometrical interpolation shape-preserving 4-point schemes

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We present several non-linear 4-point interpolation schemes, derived from the “classical” linear 4-point scheme. These new schemes have variable tension parameter instead of the fixed tension parameter in the linear 4-point scheme. The tension parameter is adapted locally according to the geometry of the control polygon within the 4-point stencil. This allows the schemes to remain local and in the same time to achieve important shape-preserving properties, such as artifacts elimination and convexity preservation. The proposed schemes are robust and have the special features of “double-knot” edges corresponding to continuity without smoothness (for artifacts elimination) and inflection edges for convexity-preservation. Convergence proof is given and experimental smoothness analysis is done in details, which indicates that away from “double-knots” the limit curves are C^1 .

Subdivision as a sequence of sampled C^p surfaces

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This article deals with analysing the convergence of a subdivision scheme and the regularity of its limit surface in the vicinity of a vertex. Many works [3, 5, 2] interpret this question as follows: the mesh around but excluding the central vertex is the control polyhedron of continuous patches. At each subdivision step, a new ring of such patches fill in a part of the n-sided hole created by the virtual removal of the central vertex. The authors analyse how this iterative insertion of new rings converges and fills in completely this hole. In contrast, we use the interpretation proposed implicitly by Doo and Sabin in the first analysis of the behaviour of the limit surface [1]. Each control mesh is viewed not as the control polyhedron of a Box-Spline surface but as the sampling of a continuous surface. Thus the sequence of meshes are samplings of a sequence of continuous surfaces which converges uniformly toward the limit surface. With these samplings we may estimate successive derivatives and derive necessary conditions for the C^p-regularity of the limit surface. In the classical works, a subdivision scheme is understood as a linear map between the new rings created at each subdivision step and which successively make the hole smaller. The conditions are given on the eigenelements of this linear map : the eigenbasis functions. There is no explicit representation of these functions, defined as limit functions, except when the scheme is Box-Spline based. In contrast, our new reading of subdivision surfaces manipulates only the subdivision matrix. Hence it can be applied to any subdivision scheme, even if it is not Box-Spline based. The resulting necessary conditions on the eigenvalues and eigenvectors of this matrix have already been proposed by Sabin [4]. In this article we prove that Sabin's proposed conditions are necessary. Our analysis may be applied at any vertex of valency greater than three.

[1] D. Doo and M. A. Sabin, Analysis of the behaviour of recursive subdivision surfaces near extraordinary points. Computer Aided Design, **10** (1978), 356–360.

[2] H. Prautzsch, Smoothness of subdivision surfaces at extraordinary vertices, Adv. Comp. Math. **9** (1998), 377–389.

[3] U. Reif, A unified approach to subdivision algorithms near extraordinary points, Computer Aided Geometric Design **12** (1995), 153–174.

[4] M. A. Sabin, Eigenanalysis and Artifacts of Subdivision Curves and Surfaces, in *Tutorials on Multiresolution in Geometric Modelling*, A. Iske, E. Quak, and M. Floater (eds.), Springer (2002).

[5] D. Zorin, Stationary subdivision and multiresolution surface representations, PhD thesis, California Institute of Technology, 1997.

Reverse subdivision

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We present an algorithm to decimate a mesh that has subdivision connectivity, so that when a uniform subdivision scheme is applied to the decimated mesh, a good approximation to the original mesh is achieved. The error between the reconstructed and original meshes can be stored so that we can reconstruct the original mesh exactly. By continuing this process, we can construct a mesh hierarchy giving a multiresolution representation of the original mesh.

The algorithm naturally splits into two parts. First we detect whether the given mesh has subdivision connectivity and construct the connectivity of the coarse level mesh. For this we use the algorithm proposed by Hormann in [1]. This uses the observation that most of the vertices of a mesh with subdivision connectivity are regular and all irregular vertices are guaranteed to be vertices of the coarse level mesh. The next stage uses the inverse of the subdivision matrix of the given uniform scheme in order to construct the geometry of the coarse level vertices.

The applications for this include lossy and lossless mesh compression, multiresolution editing, and animation.

We use the specific examples of Chaikin subdivision for the univariate case and Loop subdivision for the bivariate case.

1. K. Hormann, An easy way of detecting subdivision connectivity in a triangle mesh, Tech. Report 3, Department of Computer Science 9, University of Erlangen, May 2002.

$\sqrt{5}$-subdivision

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The possibility of an $\sqrt{5}$-scheme for quad meshes was first contemplated in [ids02c], where some potential advantages of such a scheme are discussed. For example, in the regular case the x and y coordinates of the four new vertices and the old one are all distinct, see Fig. (Left). Moreover, extending the refinement rule to the irregular case is straightforward due to a combinatorial correspondence between the new vertices and the old directed edges. To see this, notice that by connecting each new vertex with the nearest old one we introduce a local rotation of the mesh around each old vertex, and these rotations are consistent in the sense that they all have the same orientation. The refined mesh is obtained if we also connect with an edge any two new vertices corresponding to opposite directed edges, see Fig. (Right).

This subdivision scheme is of particular interest as it is the simplest scheme which never maps back to the original grid geometry, if the rotation at each subdivision step is in the same direction. We thus have the ability to create multiple possible subdivision schemes depending on whether we always rotate in the same direction, alternate rotation directions on alternate steps, or alternate rotation directions in some more complex manner.

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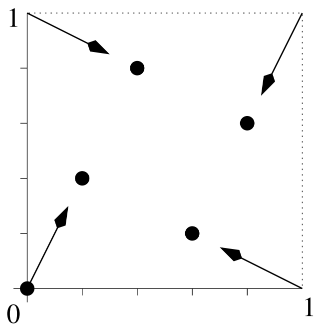

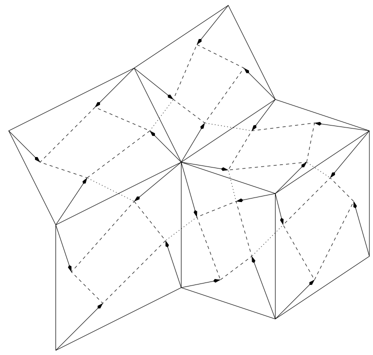

Piecewise uniform subdivision over irregular planar triangulations

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A new methodology is proposed for constructing and for analysing piecewise uniform subdivision schemes. These differ from commonly used subdivision schemes in the following aspects:

1. They operate over a (u, v) parameterized mesh.
2. They use varying subdivision weights depending on the (u, v) parameter.
3. They operate on scalar values, rather than on control points.

The subdivision is regular in the interior of each triangle, but uses different weights near edges and vertices of the original triangulation. Since the parameterization is explicit, we do not use a characteristic map for the smoothness analysis. In particular, we are able to produce C^2 functions over irregular triangulations, by a subdivision scheme which is a bivariate extension of cubic B-splines over non-uniform knot sequences.

Stable, linear spline wavelets on nonuniform knots with vanishing moments

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One of the simplest and most common approximation schemes for discrete data sets is piecewise linear interpolation on uniform grids. When this scheme is used as a basis for a multiresolution analysis it is often referred to as Faber decomposition. In an earlier paper, we studied a quadratic Hermite generalisation of Faber decomposition. In the present paper we return to Faber decomposition, but this time on nonuniform knots. Our main observation is that even in this case there exists a wavelet basis with two vanishing moments that is stable independently of the knot spacing, stability here taken with respect to an L^∞-based norm. This is in contrast to the orthogonal case where, in general, there is no wavelet basis that is stable independently of the knot spacing.

Colour-encoded mapping

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Introduction. In several interactive applications there is more information related to the geometry than displayed on the screen. To make this information available to the user, the application needs to know, at pixel accuracy, which part of the geometry the user requests information about. We propose *colour-encoded mapping* as a method for utilising graphics hardware to retrieve this information from parameterized surfaces. The method locates the parameter values (u, v) that correspond to the point of the surface in 3D, which makes it suitable to deal with objects of several levels of detail. This paper explores possibilities and implementations of this method.

Colour-encoded mapping. When rendering primitives (triangles), we may apply one or more textures, with texture coordinates associated with each vertex. The parameter domain is affinely transformed to the interval $[0, 1] * [0, 1]$, so we can use the parameter values as texture coordinates. Textures are then chosen such that by reading the final picture, the application knows exactly which part of which object resulted in each pixel.

Applications. In fly through applications the user might want to know the longitude, latitude and height above the sea at a specific point. People interact with objects in the real world to feel its shape and texture. Colour-encoded mapping has been used to generate input to a haptic device (from CompuTouch), which enables the user to feel the shape of objects.

The smoothness and approximation properties of piecewise quadratic quasi-interpolants in three variables

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Recently, we introduced a new model for the purpose of efficient visualisation of gridded volume data [RZNS03]. Our approximation method is based on quadratic splines which are defined on an appropriate uniform tetrahedral partition of the three-dimensional domain. Usage of these spaces provides several advantages concerning computational and visualisation issues which have been described in [RZNS03]. It is the purpose of this paper to analyse the mathematical properties of the new trivariate spline model. Here, we describe the smoothness properties satisfied by the quasi-interpolating trivariate splines, and prove that the approximating splines yield to nearly optimal approximation order of smooth functions. Moreover, we give some numerical tests for synthetic and real world data which verify the efficiency of our approach.

[RZNS03] Rössl, C., Zeilfelder, F., Nürnberger, G., Seidel, H.-P.: Visualisation of Volume Data with Quadratic Super Splines. In: Turk, G., Moorhead, R., van Wijk J. (ed) IEEE Visualization. Appears (2003)

Multi-scale and adaptive CS-RBFs for shape reconstruction from cloud of points

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We propose a multi-scale and adaptive method for interpolation and approximation of a point set surface by compactly supported radial basis functions. Given a set of points scattered over a surface, we first use down-sampling to construct a point set hierarchy. Then starting from the coarsest level, for each level of the hierarchy, we use compactly supported RBFs to approximate the set of points at the level as an offset of the RBF approximation computed at the previous level.

At each hierarchy level, the RBF centres are randomly chosen from the set of points of the level. The randomness is controlled by the density of points and geometric characteristic of the set. The support size of the RBF we use to approximate the point set at a vicinity of a point depends on the local density of the set at that point. Thus parts with complex geometry are approximated by dense RBFs with small supports.

Numerical experiments demonstrate high speed and good performance of the proposed method in processing irregularly sampled and/or incomplete data.

Geodesic loops on polyhedral surfaces

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Geodesic curves on polyhedral surfaces are characterised as being locally optimal at any point. From this characterisation, we have developed in [6] an iterative method to compute a geodesic path between two points : we use unfolding tools and the funnel algorithm to compute the shortest path in a 2D-face sequence (see[1], [4]). An application in a medical context (myocardium fibre modelling, see [5]) has lead us to consider geodesic loops: an extension of the method above is studied here, meaning that the endpoints correspond and that the path also satisfies the local property of geodesics at this junction. An important point to notice is that if a geodesic loop goes through no vertex of the polyhedral surface, there exists, in its neighbourhood, a geodesic loop of same length going through at least one vertex of the polyhedral surface. We will thus propose an algorithm to compute a geodesic loop on a polyhedral surface in a finite number of steps, relaxing the curve at any deviation vertex. We will show some examples. This work has some developments in the case of closed polyhedral surfaces: in [3], constrictions are detected with a careful study of the possible initialisation; in [2], geodesic loops are used to cut the surface and get the polygonal scheme.

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Classifying univariate stationary refinement schemes and predicting their behaviour

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Until a few years ago all the work in the area of univariate stationary refinement schemes was limited to consider just binary scenarios. Recent proposals of ternary subdivision schemes [1, 3, 4, 5] have introduced new interesting animals in the subdivision zoo, showing the possibility of treating refinement schemes with arity other than two. This freedom opens up a wide range of possible curves which can be classified according to the even-oddness of the width of the mask and of the arity of the refinement [2].

Since we limit ourselves to analyse only stationary refinements, all the information about a scheme is coded in its mask and its arity, which together determine the subdivision matrix and implicitly its limit function. Further investigation in spectral analysis of the subdivision matrix led to discovery of a very general necessary and sufficient criterion for predicting the behaviour of the limit function, which can also be used in the construction of new schemes with prescribed properties. Such a criterion, which extends Warren's work [8] to arity-a univariate stationary refinement schemes, gives new insights to well-known subdivision schemes, showing that we can predict the smoothness of the limit function just exploiting eigenanalysis of the subdivision matrix.

Although this extensive approach works efficiently for irregularly spaced knot sequences, in the regular case it can be remarkably simplified, thanks to the even and odd partitioning device [7]. Since the true purpose of this paper is to lay the groundwork for predicting the behaviour of more interesting bivariate subdivision schemes, we show that analysing the subdivision matrix obtained via the even and odd partitioning device, is exactly the same of treating the refinement rules in the frequency domain rather than in the spatial domain.

Additionally, in order to make univariate subdivision a special case of the bivariate one, we underline the possibility of generating a regular univariate subdivision scheme with "extraordinary point"and we show that the conditions we derived in the univariate case to produce a C^1-continuous limit function, correspond exactly to Reif's requirement of regularity and injectivity of the characteristic map [6].

This gives us hope that the conditions derived for C^2-continuity can be analogously extended to the bivariate case.

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Piecewise constant wavelets on arbitrary triangulations over the sphere

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In [1], a scalar product $\langle \cdot, \cdot \rangle_\Omega$ on $L^2(\mathbb{S}^2)$, associated to a polyhedron Ω with the vertices situated on the unit sphere $\mathbb{S}^2$, was defined. This scalar product induces a norm $\|\cdot\|_*$ equivalent to the usual 2-norm of $L^2(\mathbb{S}^2)$. In this paper we construct two classes of piecewise constant wavelets which are orthogonal with respect to this scalar product. The equivalence of the norms $\|\cdot\|_*$ and the usual norm of $L^2(\mathbb{S}^2)$ will help us to prove the Riesz stability in $L^2(\mathbb{S}^2)$ of our wavelets.

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Stability of B-wavelets

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We construct so-called minimally supported B-wavelets for a given spline order d and two nested knot sequences.

One of the important properties of B-splines is their stability. For properly normalised B-splines the stability constant is independent of the underlying knot sequence [deB73].

An interesting question here is whether we have a similar property for the set of B-wavelets. For B-wavelets of order 2, i.e. piecewise linear prewavelets of smallest support, over nonuniform knot sequences the corresponding stability constants cannot be made independent of the knot sequences involved, regardless which normalisation is chosen [OQ01].

In this talk we will present some examples and analyse how B-wavelets will act when we have order $d = 3$ and $d = 4$ on different knot sequences.

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Topology preserving thinning of vector fields on triangular meshes

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We consider the topology of piecewise linear vector fields whose domain is a piecewise linear 2-manifold, i.e. a triangular mesh. Such vector fields can describe simulated 2-dimensional flows, or they may reflect geometric properties of the underlying mesh. We introduce a thinning technique which preserves the complete topology of the vector field, i.e. the critical points and separatrices. As the theoretical foundation, we show that for local modifications of a vector field, it is possible to decide entirely by a local analysis whether or not the global topology is preserved. This result is applied in the compression algorithm which is based on a repeated local modification of the vector field — namely a repeated edge-collapse of the underlying piecewise linear domain.

Variations of angle based flattening

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Surface parameterization is an essential technique in geometry processing. Recently, methods based on the angles of a mesh have been introduced. They offer the advantage of controlling the geometric validity of the flat mesh. In a classical setting this angle based flattening aims at preserving conformality while respecting a set of angular constraints. As an alternative, we propose variations of the geometric quantity to be preserved while maintaining the validity constraints necessary to produce a parameterization. In the field of mesh optimisation several criteria have been established to quantify the quality of simplicial elements of a mesh. In general, a quality criterion is a ratio of geometric quantities such as area, edge length and radii of relevant circles. We investigate these quality criteria using the standard angle constraints in combination with new metrics in order to generate valid parameterizations with minimum distortion with respect to the quality criteria.