# The Lengauer Tarjan Algorithm for Computing the Immediate Dominator Tree of a Flowgraph

*by*

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Revised: Mon Jul 17 12:59:27 BST 2017

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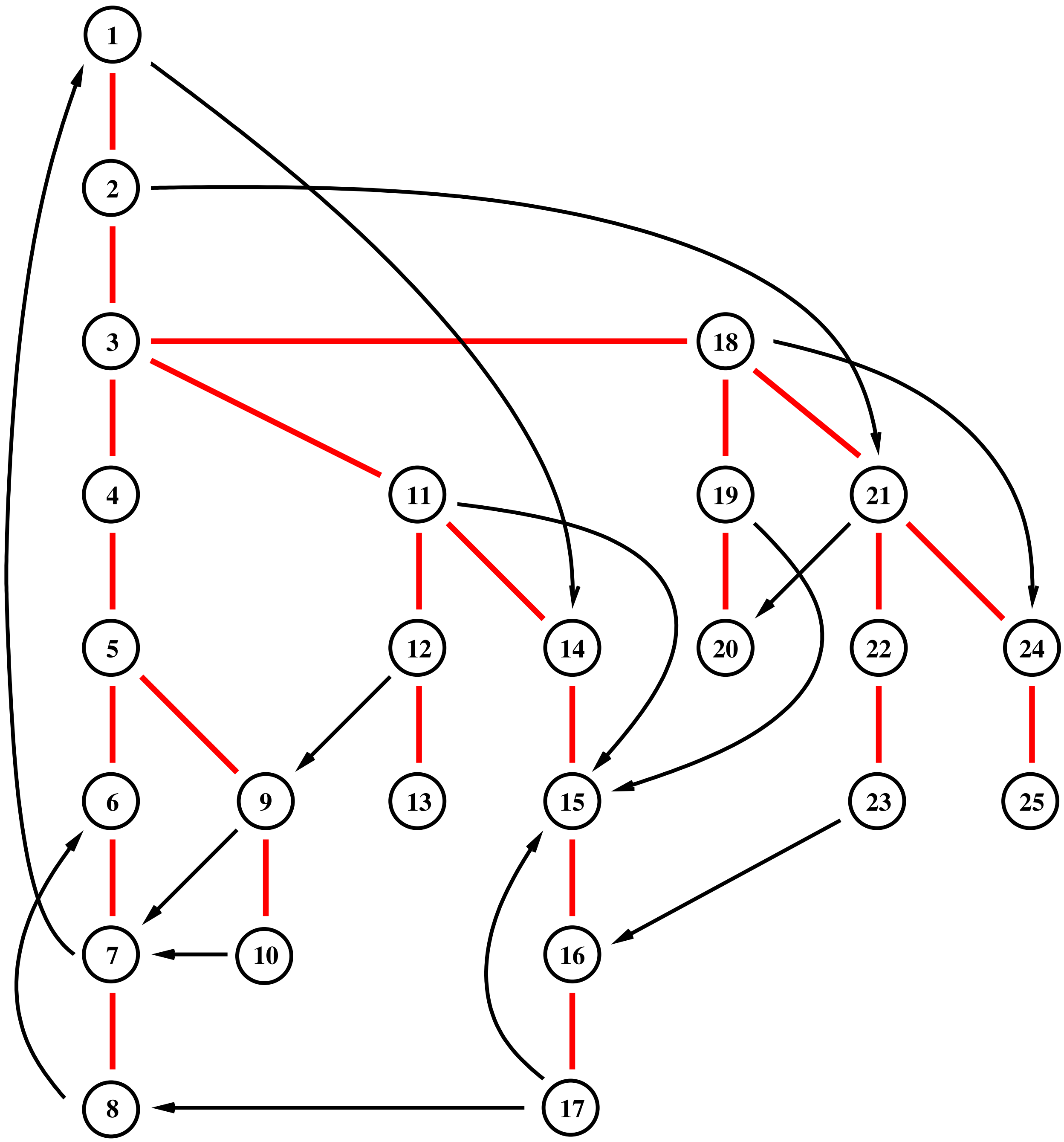
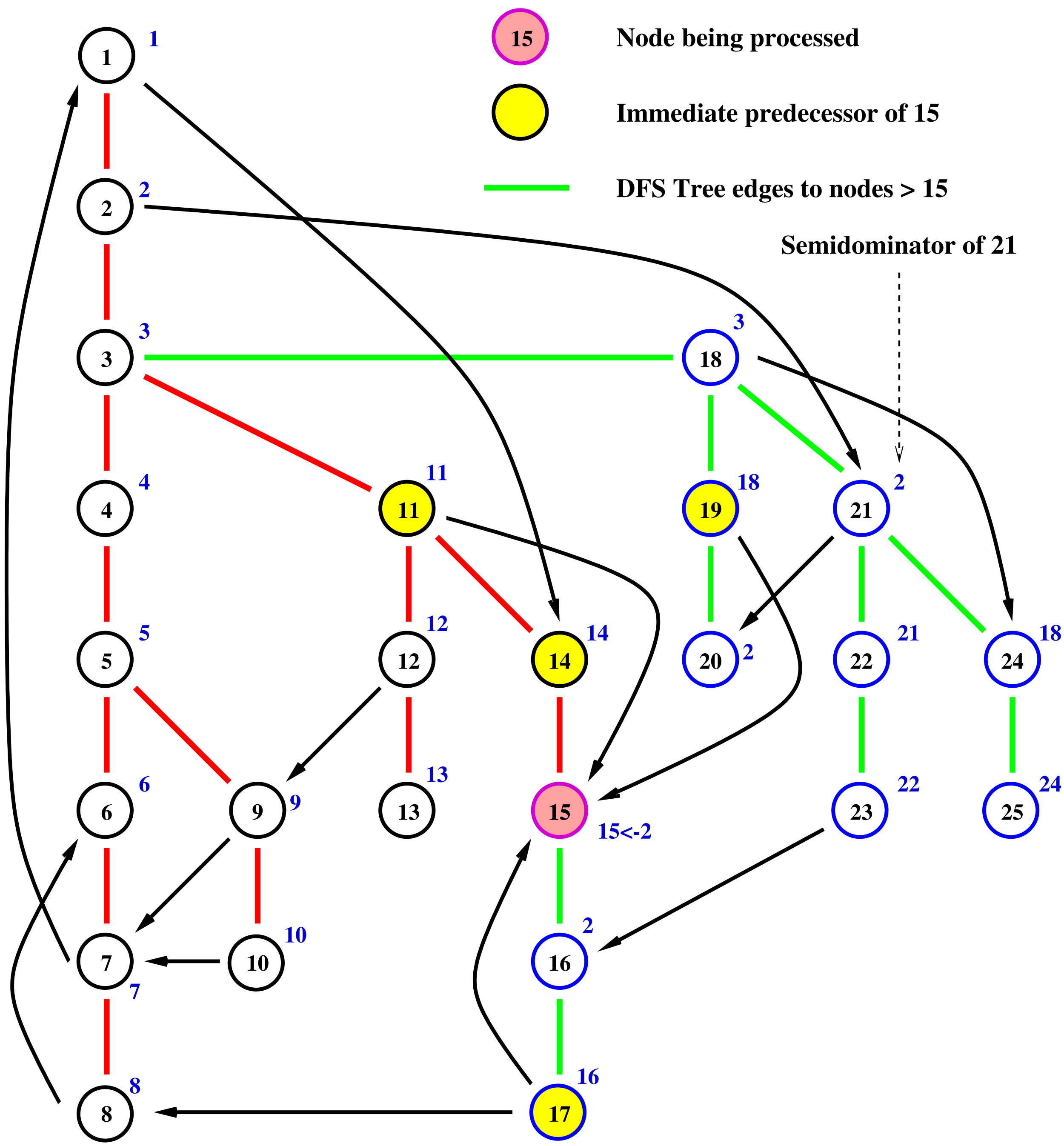
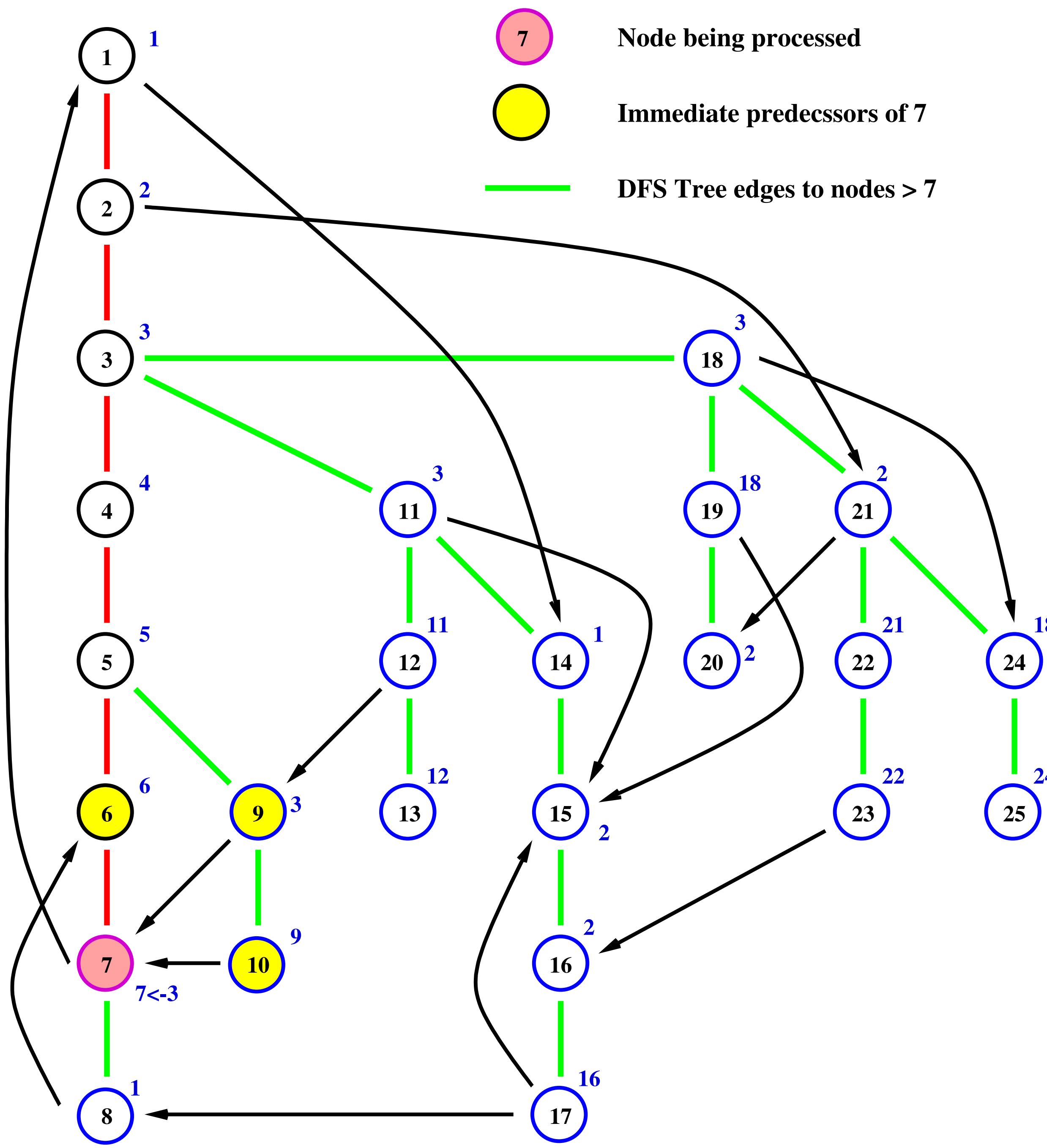
## DFS of the Flowgraph



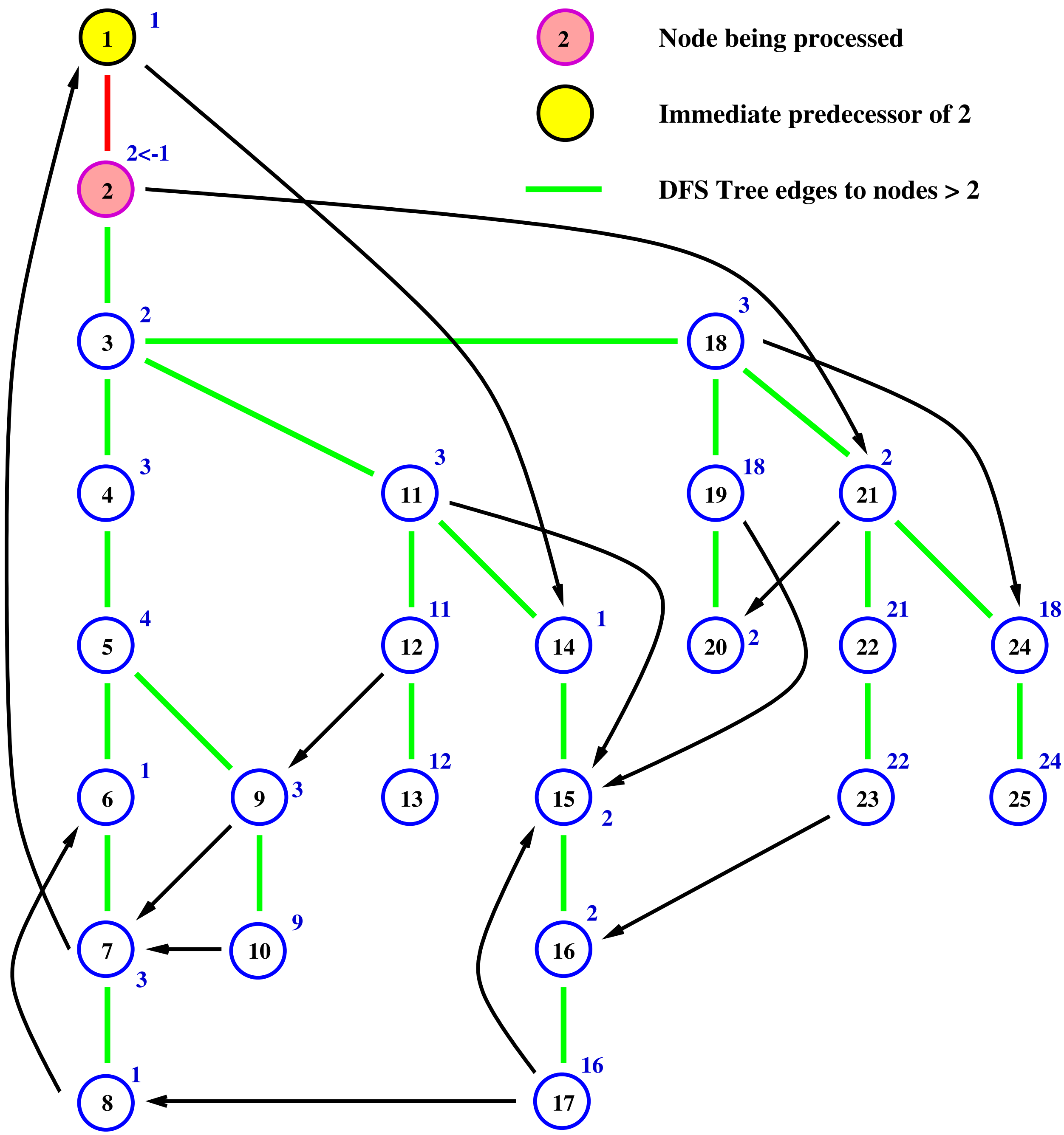
# Processing Node 15



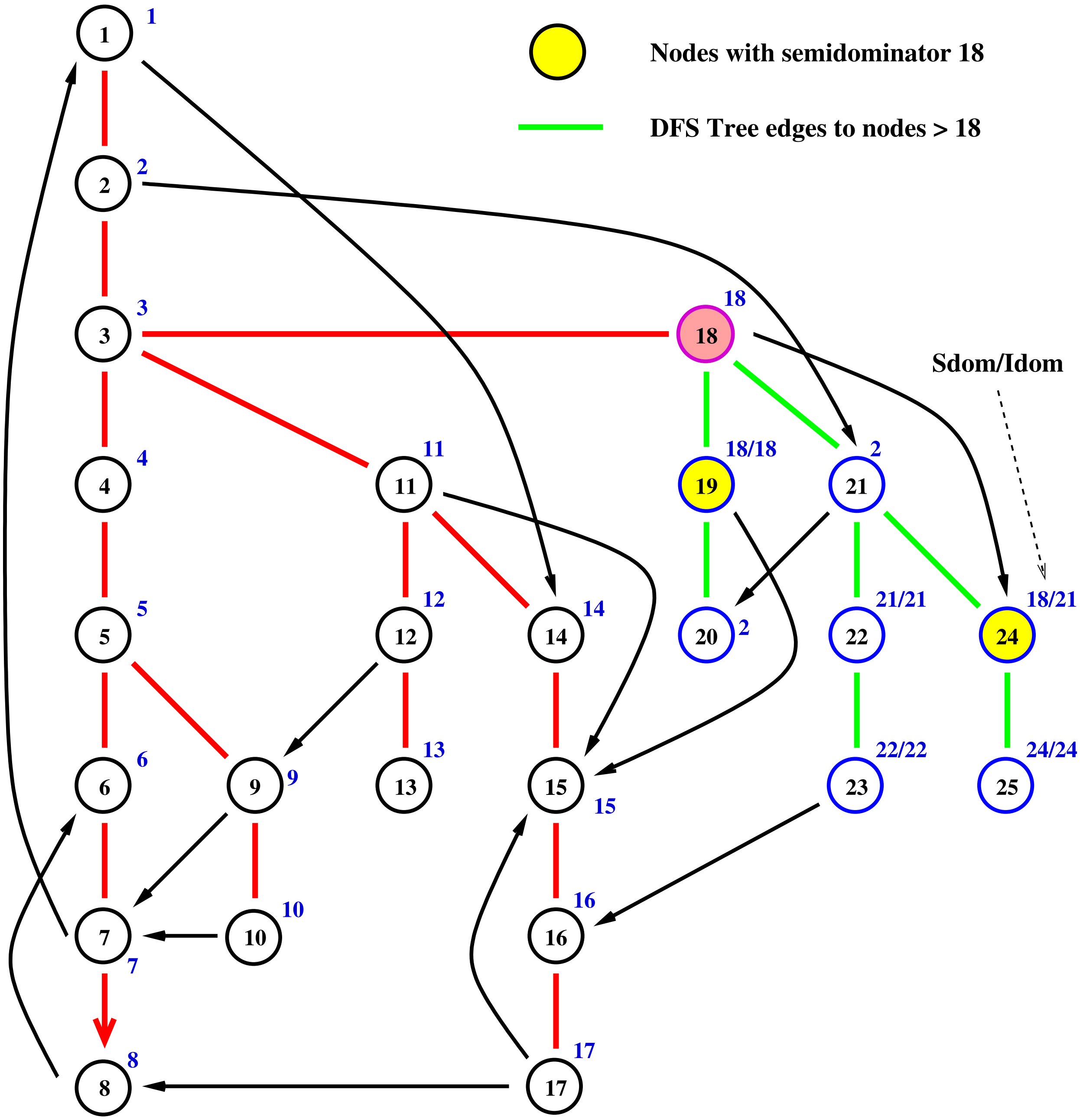
# Processing Node 7



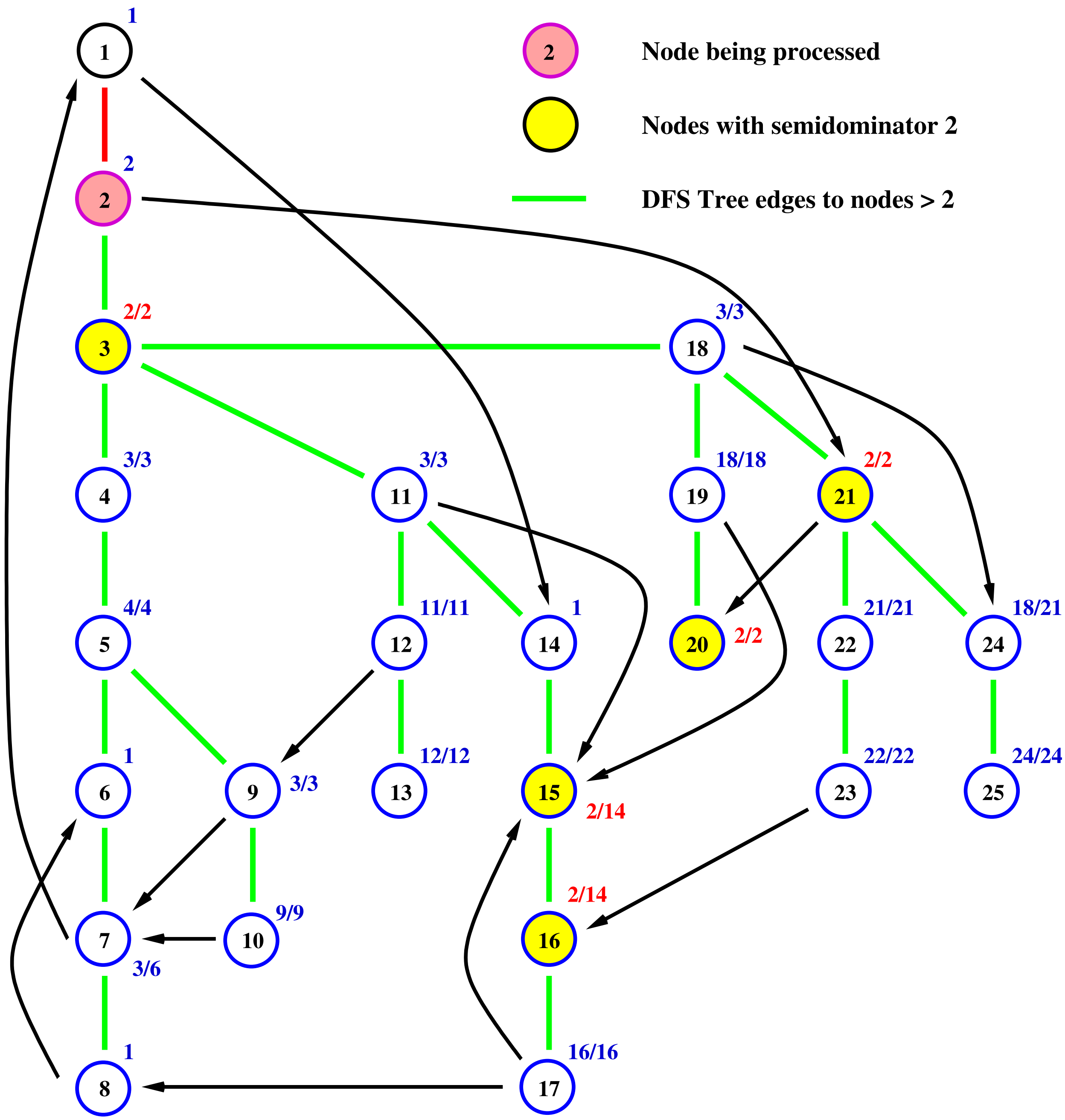
# Processing Node 2



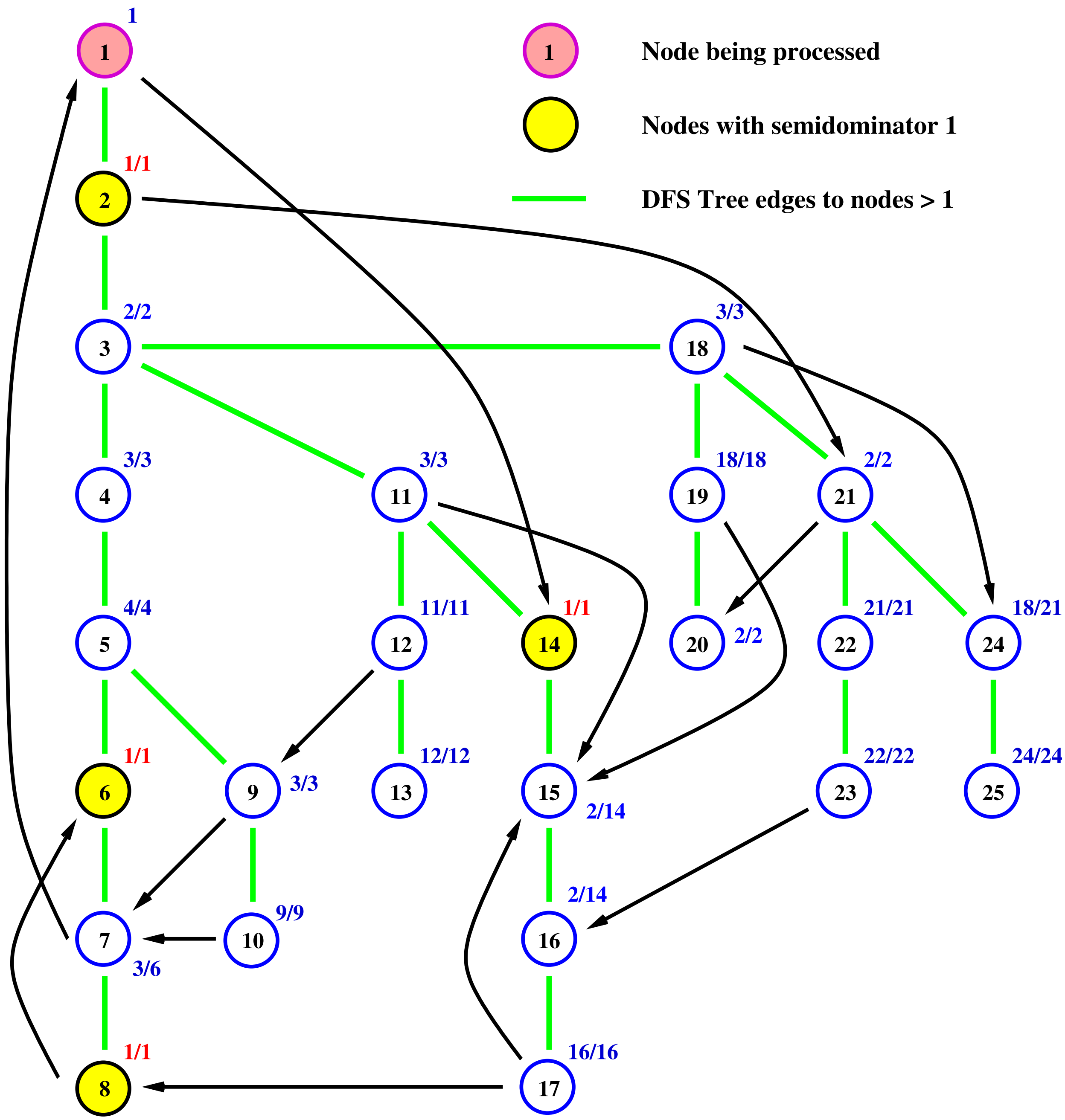
# Nodes with Semidominator 18



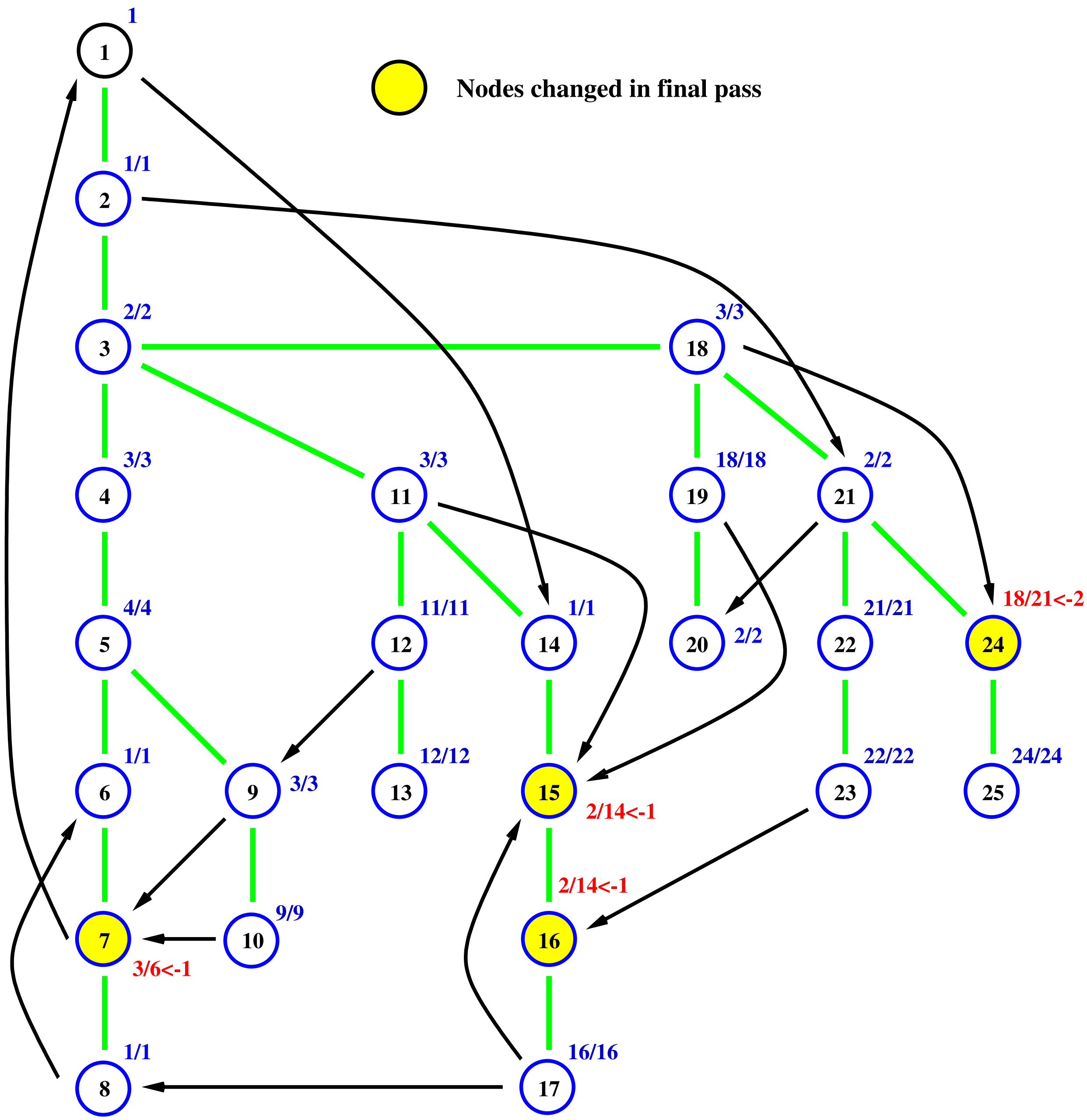
# Nodes with Semidominator 2



# Nodes with Semidominator 1



# Final Phase



Nodes changed in final pass
1
1/1
2/2
3/3
4/4
1/1
3/6<-1
1/1
3/3
9/9
11/11
12/12
1/1
2/14<-1
2/14<-1
16/16
18/18
2/2
2/2
21/21
22/22
18/21<-2
24/24

## Step 1: Initialisation

Vertices in depth first search discovery order from 1 to `n`.

For each vertex v from 1 to `n` set:

```
parent[v]     :=  DFS tree parent of v
succs[v]      :=  the given list of successors
preds[v]      :=  list of predecessors

semi[v]       :=  v
idom[v]       :=  0

ancestor[v]   :=  0
best[v]       :=  v
bucket[v]     :=  0
```

## Steps 2, 3 and 4

```
FOR w = n TO 2 BY -1 DO
{       LET p = parent[w]

step2: FOR each v in preds[w] DO
       { LET u = EVAL(v)
         IF semi[w] >  semi[u] DO
            semi[w] := semi[u]
       }
       add w to bucket[semi[w]]
       LINK(p, w)

step3: FOR each v in bucket[p]
       { LET u = EVAL(v)
         idom[v] := semi[u]<semi[v] -> u, p
       }
       bucket[p] := 0
}

step4: FOR w = 2 TO n DO
         UNLESS idom[w] = semi[w] DO
                idom[w] := idom[idom[w]]
       idom[1] := 0
```

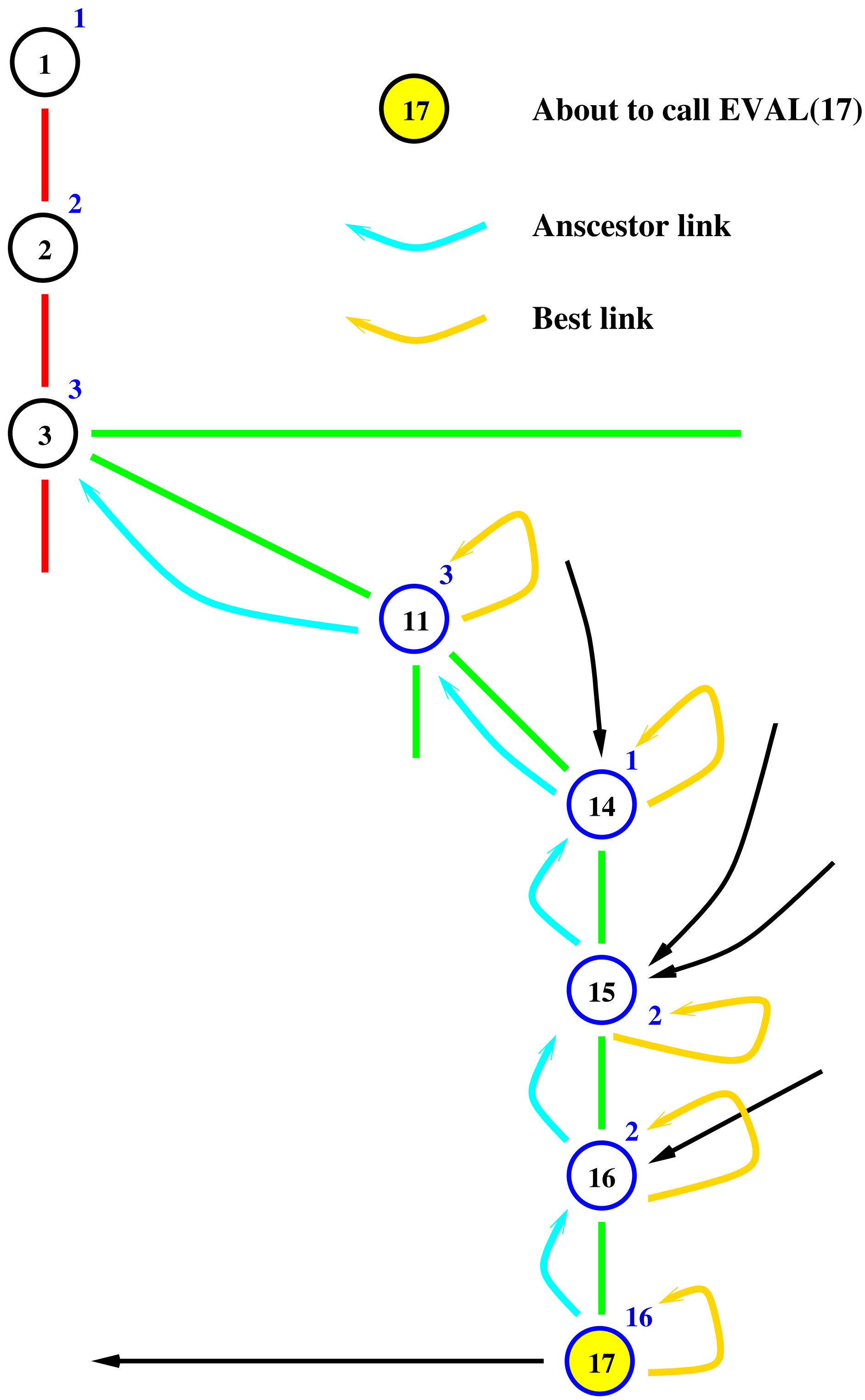
## Very Simple LINK and EVAL

```
LET LINK(v, w) BE ancestor[w] := v

LET EVAL(v) = VALOF
{ LET a = ancestor[v]

  WHILE ancestor[a] DO
  { IF semi[v] > semi[a] DO v := a
    a := ancestor[a]
  }

  // v is now the vertex
  //   with earliest semidominator
  //   of any in the ancestor chain.

  RESULTIS v
}
```

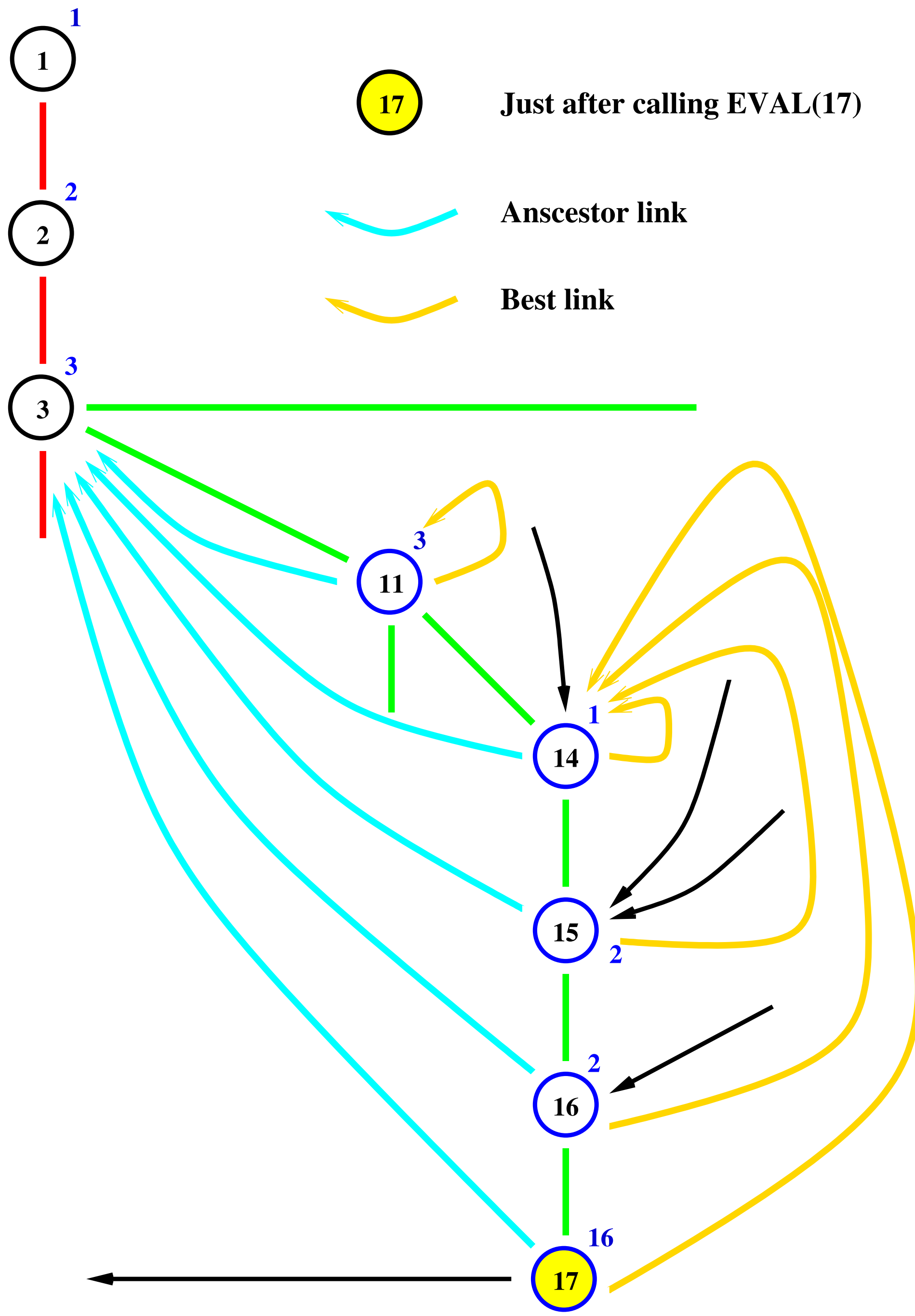
## Simple LINK and EVAL

```
LET LINK(v, w) BE ancestor[w] := v

LET EVAL(v) = VALOF
{ UNLESS ancestor[v] RESULTIS v
  COMPRESS(v)
  RESULTIS best[v]
}

AND COMPRESS(v) BE
{ LET a = ancestor[v]

  UNLESS ancestor[a] RETURN

  COMPRESS(a)

  IF semi[best[v]]  >  semi[best[a]] DO
          best[v]   :=      best[a]

  ancestor[v] := ancestor[a]
}
```

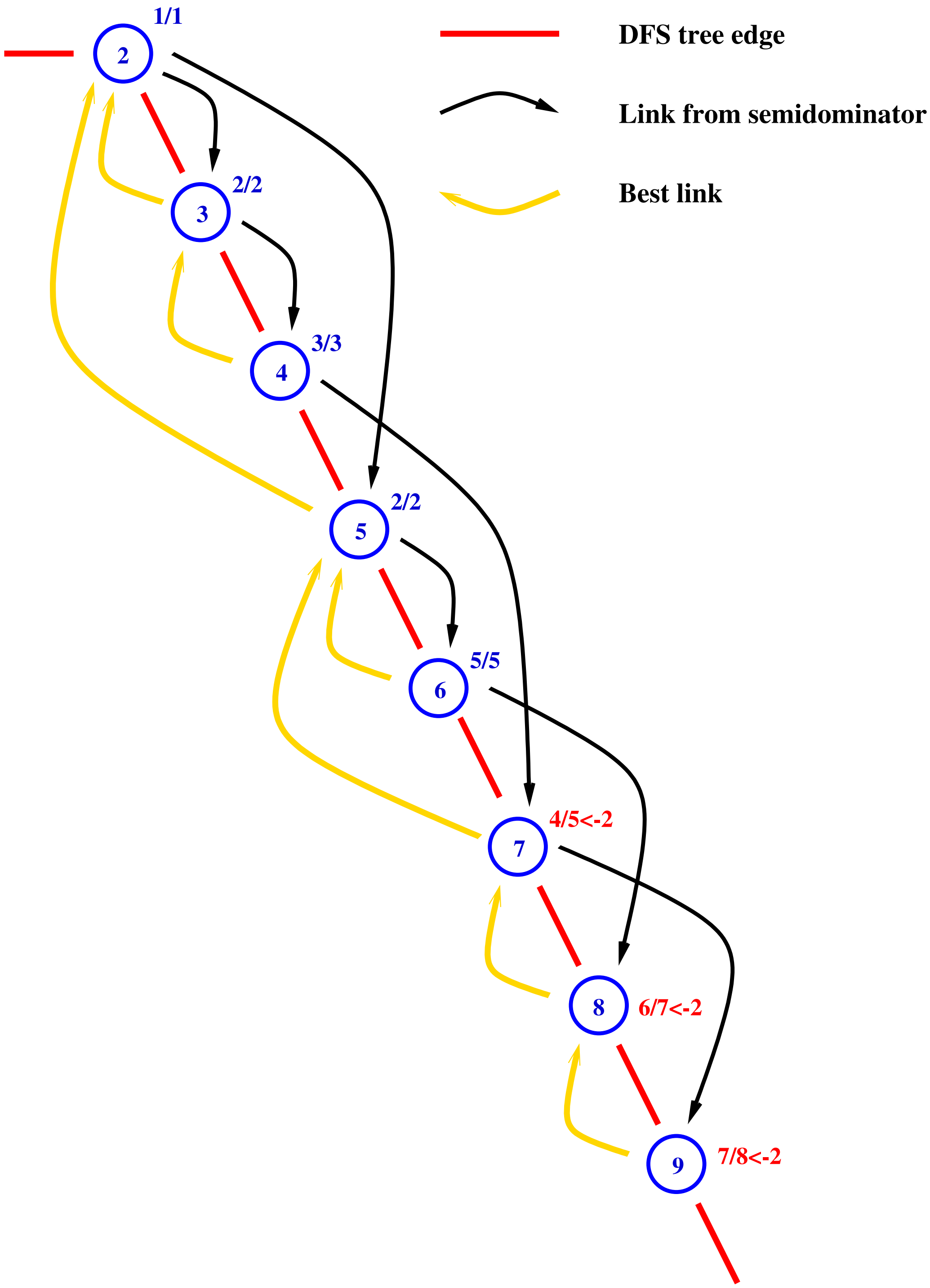
## Before calling EVAL(17)



## After calling EVAL(17)



# Step 4 Example



## Sophisticated EVAL

This version of EVAL calls COMPRESS to perform the following optimisation of the ancestor chain wherever possible.

If there is an ancestor link from x to y and one from y to z, then the link from x to y is replaced by one from x to z updating the Best field of x if necessary.

The effect of this optimisation is to modify the ancestor links so that the ancestor chain length is less than 2 for every wertex in the original chain. This clearly increases the efficiency of later calls of EVAL.

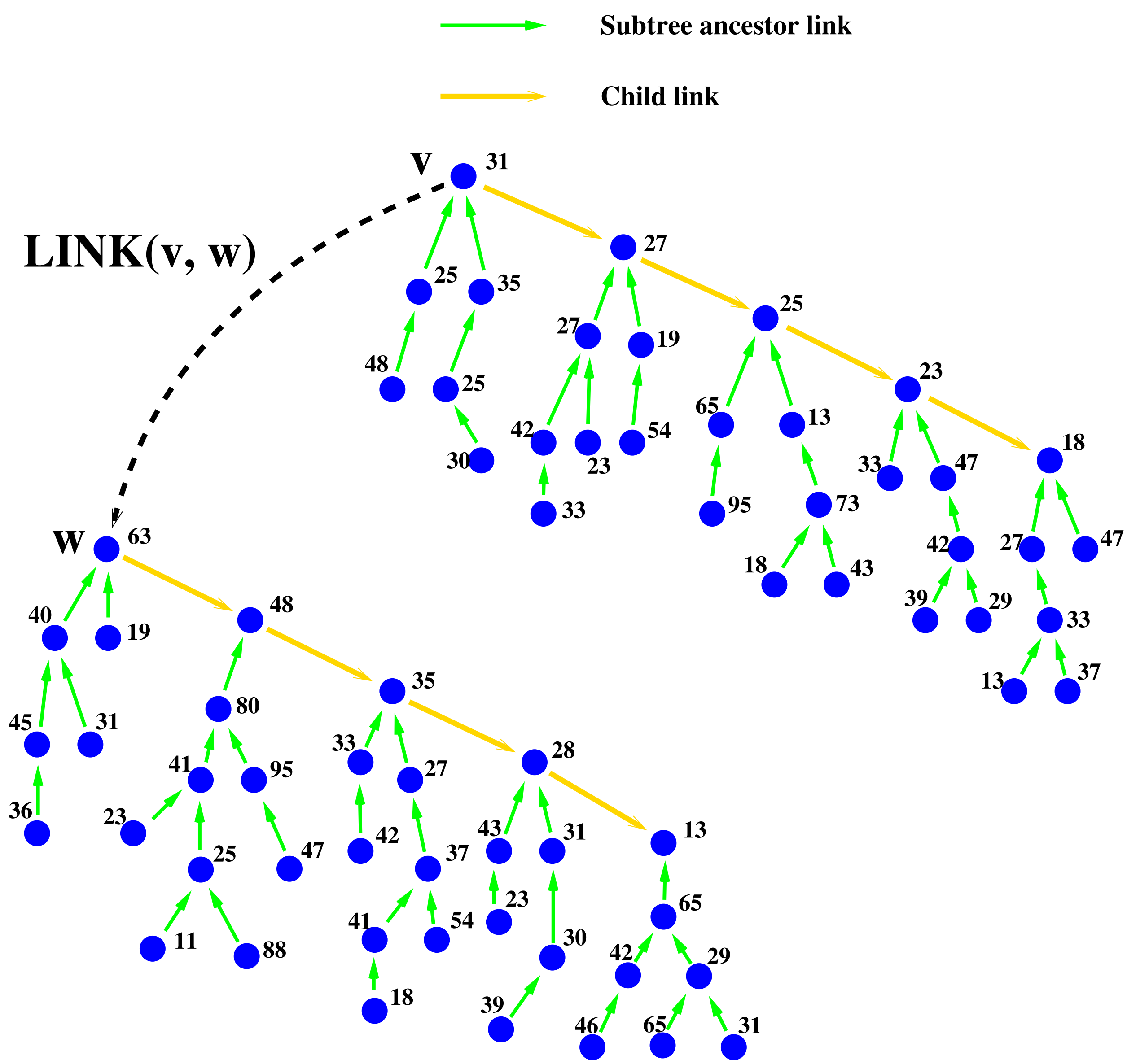
```
LET EVAL(v) = VALOF TEST Ancestor!v
THEN { RESULTIS Best!v
     }
ELSE { COMPRESS(v)
       RESULTIS Semi!(Best!(Ancestor!v)) < Semi!(Best!v) ->
                      Best!(Ancestor!v),          Best!v
     }
```

## Sophisticated LINK

```
LET LINK(v, w) BE
{ LET s = w

  { LET cs  = Child!s            // cs   = child(s)
    LET bcs = cs -> Best!cs, 0   // bcs  = best(child(s))

    TEST cs &
         Semi!(Best!w) < Semi!bcs        // bcs=0 only when cs=0
    THEN { // Combine the first two trees in the child chain,
           // making the larger one the combined root.
           LET ccs  = Child!cs           // ccs  = child(child(s))
           LET ss   = Size!s             // sc   = size(s)
           LET scs  = Size!cs            // scs  = size(child(s)
           LET sccs = ccs -> Size!ccs, 0 // sccs = size(child(child(s))
           TEST ss-scs >= scs-sccs // Compare first two tree sizes.
           THEN { Ancestor!cs := s // The first is larger or equal.
                  Child!s := ccs
           ELSE { Size!cs := ss    // The second is larger.
                  Ancestor!s := cs
                  s := cs
                }
         }
    ELSE { BREAK }
  } REPEAT

  // Now combine the two forests giving the combination the
  // child chain of the smaller forest. The other child chain is
  // then collapsed, giving all its trees ancestor links to v.
  Best!s := Best!w
  IF Size!v < Size!w DO { LET t = s; s := Child!v; Child!v := t }
  Size!v := Size!v + Size!w
  WHILE s DO { Ancestor!s := v; s := Child!s }
}
```

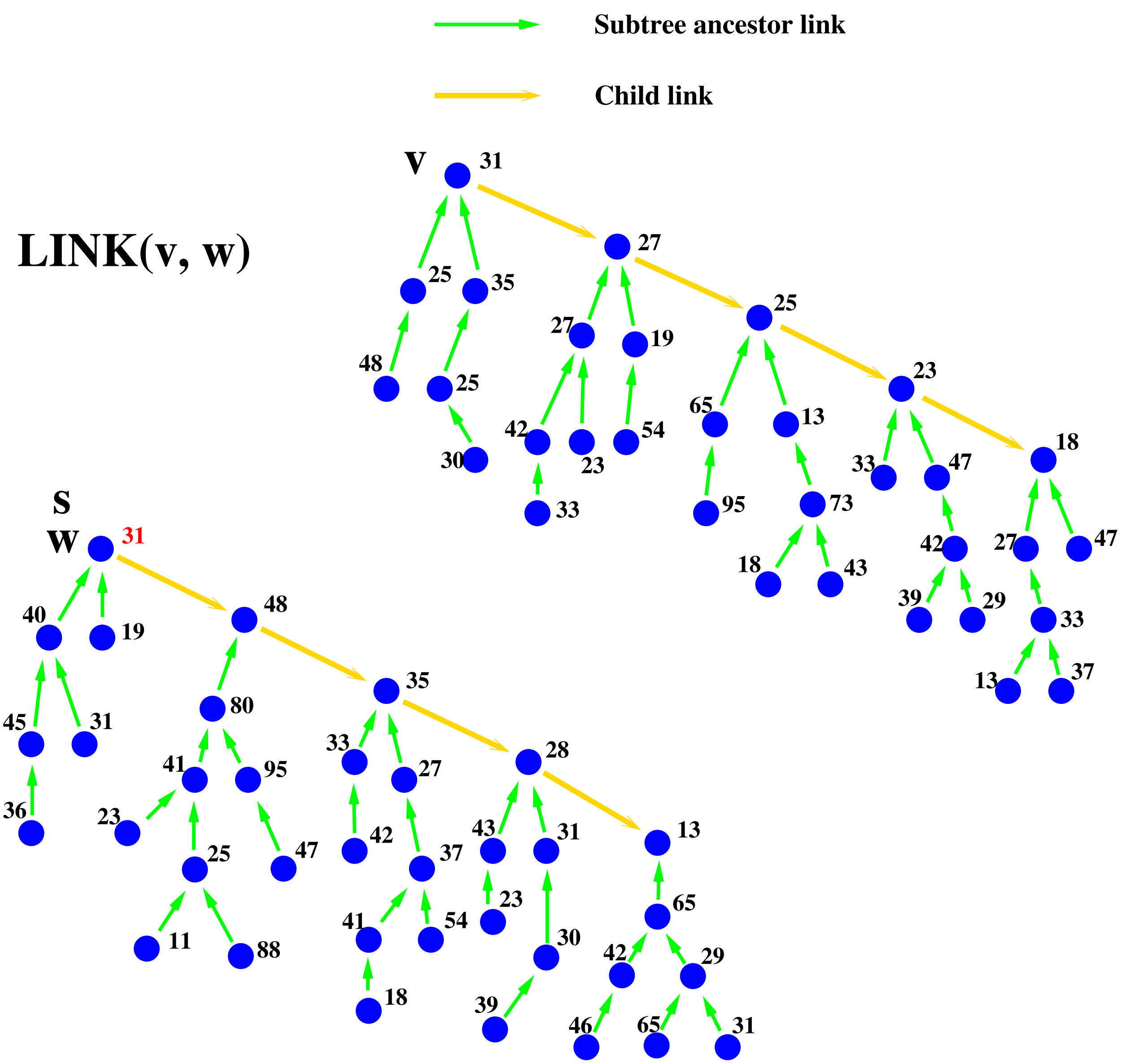
## Balanced Trees

The number next to each vertex v in the following diagram is semi!(Best!v) and, in the child chains, these are non decreasing. Note that child links are like reversed ancestor links but with this monotonicity property.

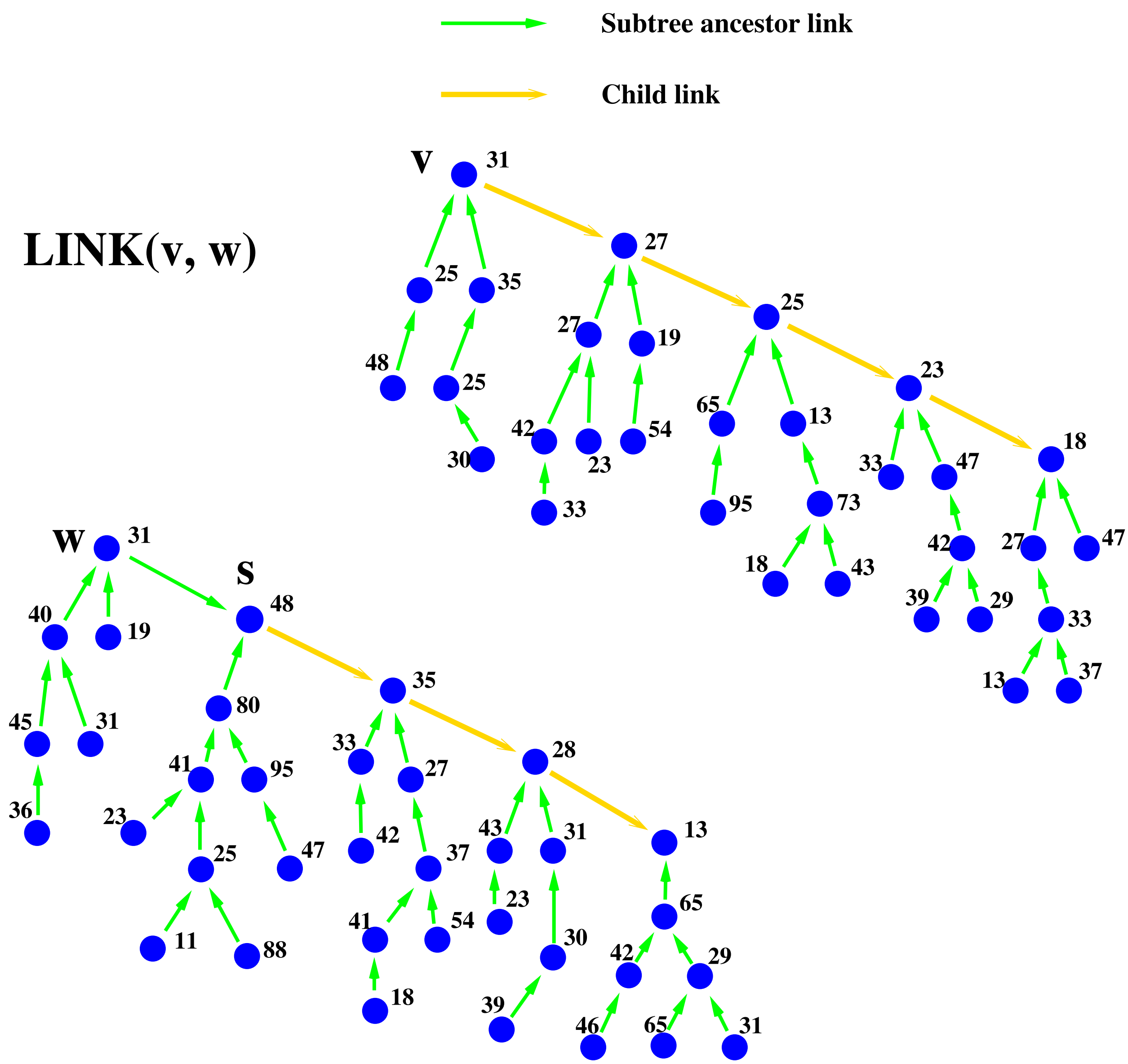


# Balanced Trees 1

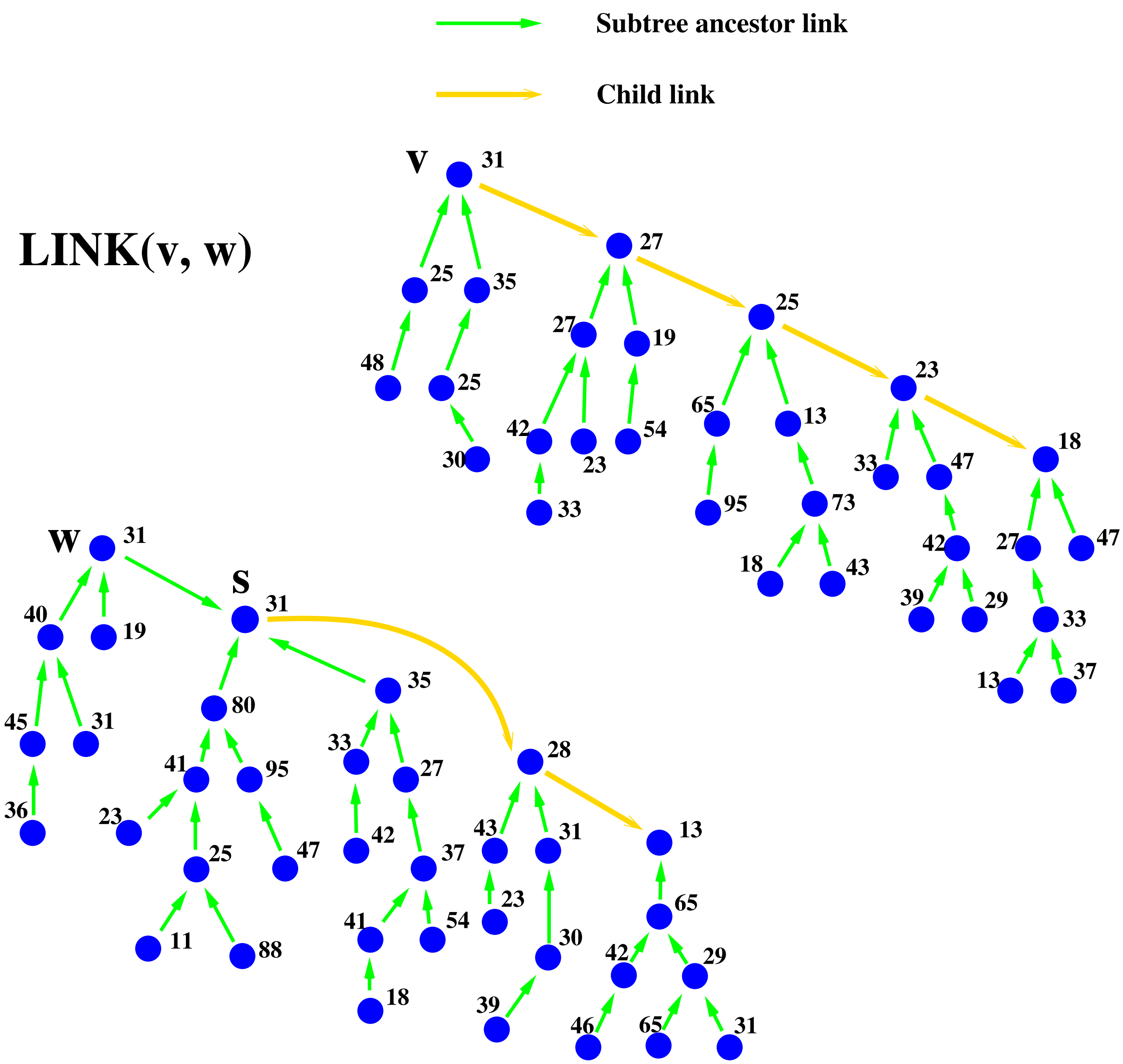
Just before LINK is called the best value of q may have been reduced possibly requiring its child chain to be modified to reinstate its monotonicity.



# Balanced Trees 2

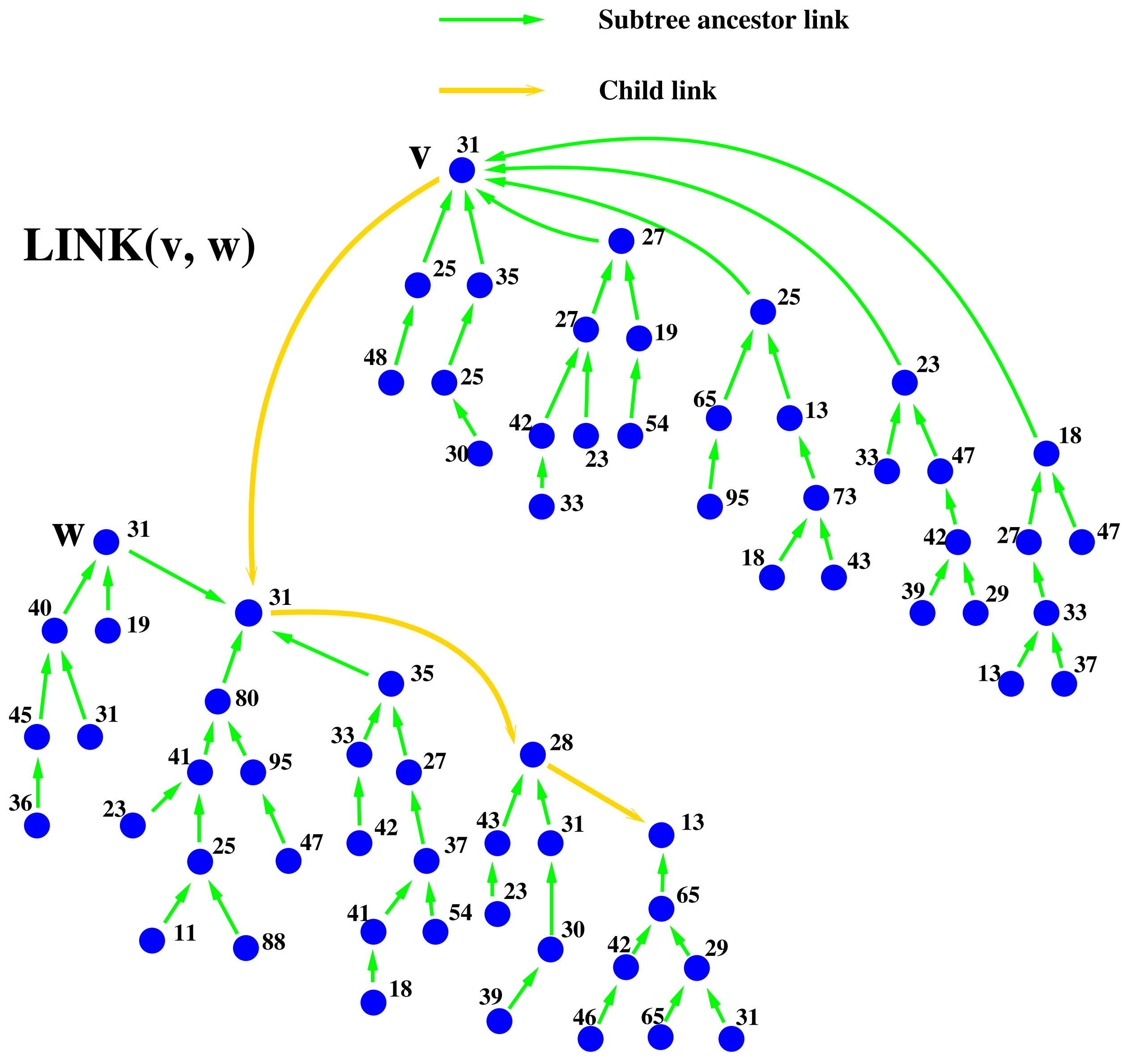


# Balanced Trees 3



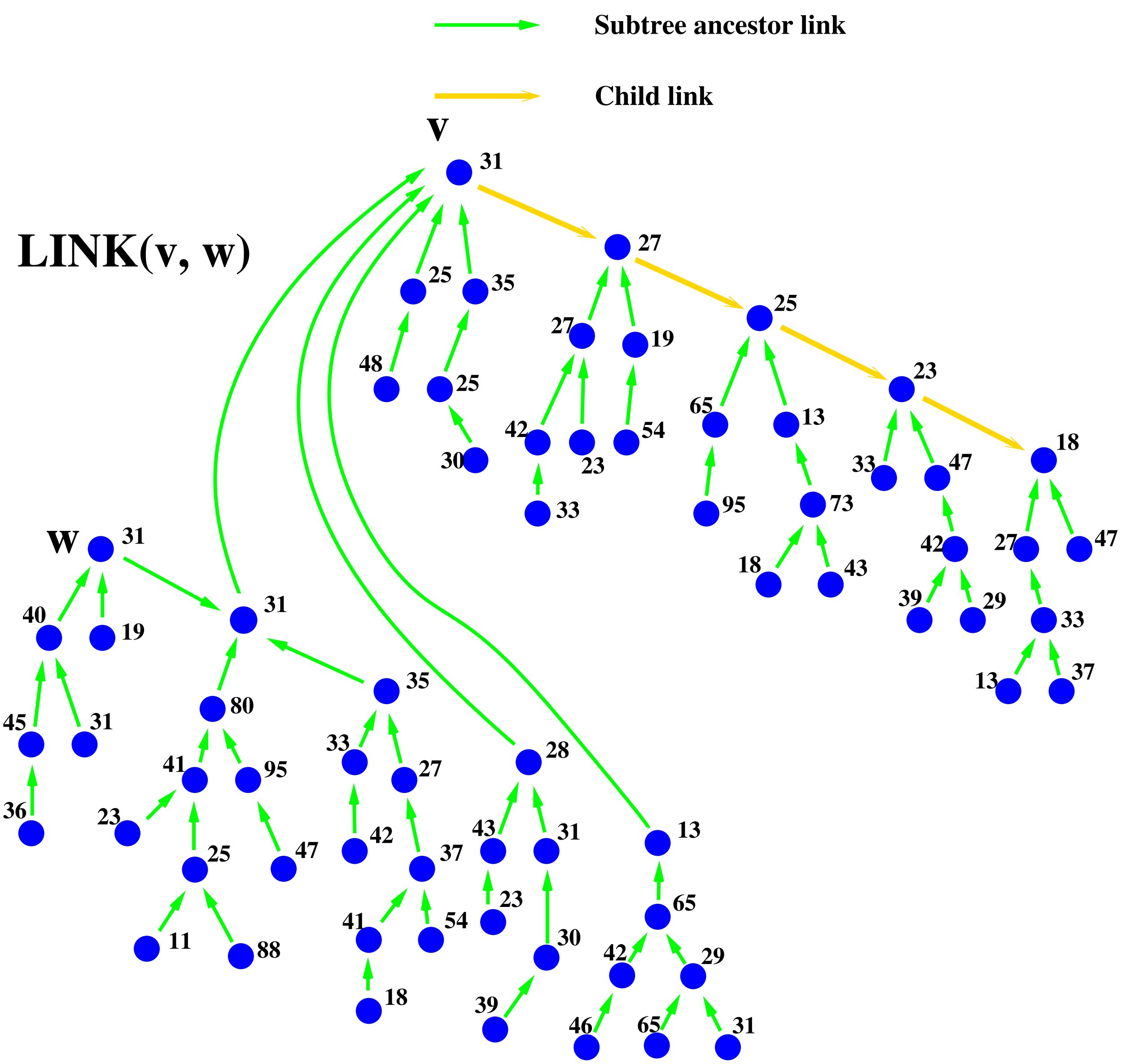
# Balanced Trees 4a

The following is the result if the forest rooted at v had fewer vertices than the forest that was rooted at w.



# Balanced Trees 4b

The following is the result if the forest rooted at v had the same number or more vertices than the forest that was rooted at w.



## Experimental Results

Results from running the BCPL program `bcplprogs/dom/lt.b` which applies the three variants of the algorithm to random graphs.

| Random Graph | | | Cintcode Instruction Counts | | |
|---|---|---|---|---|---|
| Nodes | Edges | Seed | v.simple | simple | sophisticated |
| 1000 | 1500 | 1 | 284819 | 272331 | 321589 |
| 1000 | 2000 | 1 | 455097 | 317233 | 358262 |
| 1000 | 2500 | 1 | 1180722 | 376698 | 388822 |
| 1000 | 3000 | 1 | 2440849 | 445055 | 416713 |
| 1000 | 5000 | 1 | 5680947 | 630373 | 542251 |
| 1000 | 10000 | 1 | 12848479 | 1049128 | 850692 |
| 10000 | 50000 | 1 | 334315826 | 6614589 | 5432695 |
| 10000 | 100000 | 1 | 949583241 | 10928097 | 8541841 |
| 100000 | 400000 | 1 | - | 56932784 | 48589754 |
| 100000 | 123289 | 1 f | 24592148 | 25380561 | 32042239 |