Mathematical Synthesis of Equational Deduction Systems

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Our view of computation is still evolving. The *concrete theories* for specific computational phenomena that are emerging encompass three aspects: specification and programming languages for describing computations, mathematical structures for modelling computations, and logics for reasoning about properties of computations. To make sense of this complexity, and also to compare and/or relate different concrete theories, *meta-theories* have been built. These meta-theories are used for the study, formalisation, specification, prototyping, and testing of concrete theories.

Our main concern here is the investigation of meta-theories to provide systems that better support the formalisation of concrete theories. Thereby we propose a research programme based on the development of mathematical models of computational languages, and the systematic use of these models to synthesise formal deduction systems for reasoning and computation. Specifically, we put forth a mathematical methodology for the synthesis of equational and rewriting logics from algebraic meta-theories. The synthesised logics are guaranteed to be sound with respect to a canonical model theory, and we provide a framework for analysing completeness that typically leads to canonical logics.

Our methodology can be used to rationally reconstruct the traditional equational logic of universal algebra and its multi-sorted version from first principles. As for modern applications, we have synthesised: (1) a nominal equational logic for specifying and reasoning about languages with name-binding operators, and (2) a second-order equational logic for specifying and reasoning about simple type theories. Overall, we aim at incorporating into the research programme further key features of modern languages, as *e.g.* type dependency, linearity, sharing, and graphical structure.

References

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