Extensible proof-producing compilation

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This talk is about compiling functions from the HOL4 theorem prover to machine code.
Motivation

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What is HOL4?

- an interactive and programmable proof assistant
- implements higher-order logic
- used for formalising maths, verification of hardware and software ... (e.g. Anthony Fox has used it for verifying the hardware of an ARM processor)
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Aim: user verifies an algorithm, clicks a button and then receives machine code, which is guaranteed (via proof in HOL4) to correctly implement the algorithm.
Example

Given function $f$ as input

$$f(r_1) = \begin{cases} r_1 & \text{if } r_1 < 10 \\ r_1 - 10 & \text{else} \end{cases}$$

the compiler generates ARM machine code:

```
E351000A  L:   cmp r1,#10
2241100A   subcs r1,r1,#10
2AFFFFFFFC  bcs L
```
Example

Given function $f$ as input

$$f(r_1) = \text{if } r_1 < 10 \text{ then } r_1 \text{ else let } r_1 = r_1 - 10 \text{ in } f(r_1)$$

the compiler generates ARM machine code:

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```

and automatically proves a certificate HOL4 theorem, which states that $f$ is executed by machine code:

$$\vdash \{ r_1 \ r_1 \text{* pc } p \text{* s} \}$$

$$p : E351000A \ 2241100A \ 2AFFFFFFFC$$

$$\{ r_1 \ f(r_1) \text{* pc } (p+12) \text{* s} \}$$
Example, cont.

One can prove properties of $f$ since it lives in HOL4:

\[ \vdash \forall x. f(x) = x \mod 10 \]

Here mod is modulus over unsigned machine words.
Example, cont.

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Properties proved of $f$ translate to properties of the machine code:

\[ \vdash \{ r1 \ r1 * pc \ p * s \} \]

\[ p : \text{E351000A 2241100A 2AFFFFFFC} \]
\[ \{ r1 \ (r1 \mod 10) * pc \ (p+12) * s \} \]
Example, cont.

One can prove properties of $f$ since it lives in HOL4:

$$\forall x. \ f(x) = x \mod 10$$

Here mod is modulus over unsigned machine words.

Properties proved of $f$ translate to properties of the machine code:

$$\forall \ p \ r1 \ r1_r1 *, pc p *, s \}
\{ \ r1 \ (r1 \ mod \ 10) * pc (p + 12) * s \}

Additional feature: the compiler can use the above theorem to extend its input language with: let $r1 = r1 \mod 10$ in
Talk outline

1. how is the proof-producing compiler implemented?
2. how do extensions work? example: LISP interpreter
3. design decisions and related work
Methodology

To compile function $f$:

1. **code generation:**
   
   generate, without proof, machine code from input $f$;

2. **decompilation:**
   
   derive, via proof, a function $f'$ describing the machine code;

3. **certification:**
   
   prove $f = f'$.

In TACAS’98, Pnueli et al. call this method **translation validation**.
Example, code generation

When compiling function $f$:

$$f(r_0, r_1, m) =$$

if $r_0 = 0$ then $(r_0, r_1, m)$ else
let $r_1 = m(r_1)$ in
let $r_0 = r_0 - 1$ in
$f(r_0, r_1, m)$

Code generation produces x86 assembly:
Example, code generation

When compiling function $f$: 

\[
    f(r_0, r_1, m) =
    \begin{cases}
        (r_0, r_1, m) & \text{if } r_0 = 0 \\
        \text{let } r_1 = m(r_1) \text{ in} \\
        \text{let } r_0 = r_0 - 1 \text{ in} \\
        f(r_0, r_1, m)
    \end{cases}
\]

Code generation produces x86 assembly:

L1:  test eax, eax  
     jz L2  
     mov ecx,[ecx]  
     dec eax  
     jmp L1  
L2:
Example, code generation

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let $r_1 = m(r_1)$ in

let $r_0 = r_0 - 1$ in

$f(r_0, r_1, m)$

Code generation produces x86 assembly, which NASM translates:

```
0:  85C0 L1:  test eax, eax
2:  7405    jz L2
4:  8B09    mov ecx,[ecx]
6:   48     dec eax
7:  EBF7    jmp L1
L2:
```
Initial input language

The initial input language is designed for ease of code generation:

- all variables must have names of registers $r_0$, $r_1$, $r_2$, stack locations $s_1$, $s_2$, or memory functions $m$, $m_1$, $m_2$ etc.
- basic operations over registers are permitted, e.g.
  - let $r_1 = r_2 + r_4$ in ...
  - let $r_3 = 50$ in ...
- simple comparisons are supported, e.g.
  - if $(r_2 = 5) \land (r_3 \& 3 = 0)$ then ... else ...
- tail-recursive function calls allowed.

This language is very restrictive, but can be used as compiler back-end, or extended directly (see later slides).
Example, decompilation

Returning to our example... the second stage of compilation is *decompilation* of the generated code (FMCAD 2008).

Decompilation: derive a function $f'$ describing the code.
Example, decompilation

Returning to our example... the second stage of compilation is *decompilation* of the generated code (FMCAD 2008).

Decomposition: derive a function $f'$ describing the code.

First, theorems describing one pass through the code are derived:

\[
eax & eax = 0 \Rightarrow \\
\{ (eax, ecx, m) is (eax, ecx, m) * eip \ p * s \}
\]

\[
p : 85C074058B0948EBF7
\]

\[
\{ (eax, ecx, m) is (eax, ecx, m) * eip (p+9) * s \}
\]

\[
eax & eax \neq 0 \land \text{ecx} \in \text{domain } m \land (ecx & 3 = 0) \Rightarrow \\
\{ (eax, ecx, m) is (eax, ecx, m) * eip \ p * s \}
\]

\[
p : 85C074058B0948EBF7
\]

\[
\{ (eax, ecx, m) is (eax-1, m(ecx), m) * eip \ p * s \}
\]
A special loop rule is used to introduce a tail recursion.

\[ \forall \text{res res'} c. \quad (\forall x. P x \land G x \Rightarrow \{\text{res } x\} c \{\text{res } (F x)\}) \land \\
(\forall x. P x \land \neg (G x) \Rightarrow \{\text{res } x\} c \{\text{res'} (D x)\}) \Rightarrow \\
(\forall x. \text{pre } x \Rightarrow \{\text{res } x\} c \{\text{res'} (\text{tailrec } x)\}) \]

where \text{tailrec} and \text{pre} are:

\[
\text{tailrec } x = \text{if } (G x) \text{ then } \text{tailrec } (F x) \text{ else } (D x)
\]

\[
\text{pre } x = P x \land (G x \Rightarrow \text{pre } (F x))
\]
Example, decompilation, cont.

With appropriate instantiations of variables, \( \text{tailrec} \) satisfies:

\[
\text{tailrec}(eax, ecx, m) =
\begin{align*}
\text{if } eax \& eax = 0 \text{ then } (eax, ecx, m) \text{ else } \\
\text{let } ecx = m(ecx) \text{ in } \\
\text{let } eax = eax - 1 \text{ in } \\
\text{tailrec}(eax, ecx, m)
\end{align*}
\]

and we have a certificate theorem:

\[
\text{pre}(eax, ecx, m) \Rightarrow \\
\{ (eax, ecx, m) \text{ is } (eax, ecx, m) \ast \text{eip } p \ast s \} \\
p : 85C074058B0948EBF7 \\
\{ (eax, ecx, m) \text{ is } \text{tailrec}(eax, ecx, m) \ast \text{eip } (p+9) \ast s \}
\]

We define decompilation \( f' = \text{tailrec} \).
Certification

To compile function \( f \):

1. code generation:
   generate, without proof, machine code from input \( f \);

2. decompilation:
   derive, via proof, a function \( f' \) describing the machine code;

3. certification:
   prove \( f = f' \).
Example, certification

Since \( f \) and \( f' \) are instances of \texttt{tailrec},

\[
\text{tailrec } x = \text{if } ( G \ x) \text{ then } \text{tailrec } ( F \ x) \text{ else } ( D \ x)
\]

it is sufficient to prove their components equivalent, in this case:

\[
(\lambda(r_0, r_1, m). \ r_0 \neq 0) = (\lambda(eax, ecx, m). \ eax \& eax \neq 0)
\]
\[
(\lambda(r_0, r_1, m). \ (r_0-1, m(r_1), m)) = (\lambda(eax, ecx, m). \ (eax-1, m(ecx), m))
\]
\[
(\lambda(r_0, r_1, m). \ (r_0, r_1, m)) = (\lambda(eax, ecx, m). \ (eax, ecx, m))
\]

Lightweight optimisations are undone:
▶ small tweaks, like \( eax \& eax = eax \);
▶ some instruction reordering;
▶ conditional execution (for ARM and x86);
▶ dead-code removal;
▶ shared-tail elimination (next slides)
Example, certification

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it is sufficient to prove their components equivalent, in this case:

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\begin{align*}
(\lambda(r_0, r_1, m). \ r_0 \neq 0) & = (\lambda(eax, ecx, m). \ eax \ & eax \neq 0) \\
(\lambda(r_0, r_1, m). \ (r_0-1, m(r_1), m)) & = (\lambda(eax, ecx, m). \ (eax-1, m(ecx), m)) \\
(\lambda(r_0, r_1, m). \ (r_0, r_1, m)) & = (\lambda(eax, ecx, m). \ (eax, ecx, m))
\end{align*}
\]

Lightweight optimisations are undone:

- small tweaks, like \( eax \ & eax = eax \);
- some instruction reordering;
- conditional execution (for ARM and x86);
- dead-code removal;
- shared-tail elimination (next slides)
Shared-tail elimination

The assignment to $r_1$ is shared:

$$f(r_1, r_2) = \begin{cases} \text{if } r_1 = 0 \text{ then let } r_2 = 23 \text{ in let } r_1 = 4 \text{ in } (r_1, r_2) \\ \text{else let } r_2 = 56 \text{ in let } r_1 = 4 \text{ in } (r_1, r_2) \end{cases}$$

Another formulation:

$$g(r_1, r_2) = \begin{cases} \text{let } (r_1, r_2) = g_2(r_1, r_2) \text{ in let } r_1 = 4 \text{ in } (r_1, r_2) \end{cases}$$

$$g_2(r_1, r_2) = \begin{cases} \text{if } r_1 = 0 \text{ then let } r_2 = 23 \text{ in } (r_1, r_2) \\ \text{else let } r_2 = 56 \text{ in } (r_1, r_2) \end{cases}$$

Both produce ARM code:

0: E3510000  cmp r1,#0
4: 03A02017  moveq r2,#23
8: 13A02038  movne r2,#56
12: E3A01004  mov r1,#4
Talk outline

1. how to implement basic proof-producing compiler?
2. how do extensions work? LISP interpreter.
3. design decisions and related work
The introduction showed how to prove:

\[\{r_1 \cdot r_1 \mod 10 \cdot pc \cdot (p+12) \cdot s\}\]

\[p: E351000A \ 2241100A \ 2AFFFFFFC\]

Such theorems can be used to extend the compiler’s input language, in this case with:

\[
\text{let } r_1 = r_1 \mod 10 \text{ in }
\]
Example. The extension allows us to compile:

\[ f(r_1, r_2, r_3) = \text{let } r_1 = r_1 + r_2 \text{ in} \]
\[ \text{let } r_1 = r_1 + r_3 \text{ in} \]
\[ \text{let } r_1 = r_1 \mod 10 \text{ in} \]
\[ r_1 \]

Code generation produces “tagged-code”:

E0811002 E0811003 E351000A 2241100A 2AFFFFFC

The decompiler will know to use the supplied theorem for tagged code blocks. The certification stage is unchanged.
The one-pass theorem is derived using the supplied theorem:

\[
\{ r_1 \ r_1 \ast pc \ p \ast s \}
\]

\[
p : E0811002 \ E0811003 \ E351000A \ 2241100A \ 2AFFFFFC
\]

\[
\{ r_1 \ ((r_1 + r_2 + r_3) \ mod \ 10) \ast pc \ (p+20) \ast s \}
\]

Previously proved theorems are used as building blocks.
Example, LISP interpreter

Abstract extensions can also be made.

As a case study, we compiled a small LISP interpreter.

Theorems were proved for primitive LISP operations, e.g.

\[(\exists x \ y. \ v_1 = \text{Dot } x \ y) \Rightarrow \{ \text{lisp } (v_1, v_2, v_3, v_4, v_5, v_6, l) \ast \text{pc } p \} \]

\[p : \text{E5933000} \]
\[\{ \text{lisp } (\text{car } v_1, v_2, v_3, v_4, v_5, v_6, l) \ast \text{pc } (p + 4) \} \]

\[(\text{size } v_1 + \text{size } v_2 + \text{size } v_3 + \text{size } v_4 + \text{size } v_5 + \text{size } v_6) < l \Rightarrow \{ \text{lisp } (v_1, v_2, v_3, v_4, v_5, v_6, l) \ast \text{s } \ast \text{pc } p \} \]

\[p : \text{E50A3018 } \text{E50A4014 } \text{E50A5010 } \ldots \text{E51A7008 } \text{E51A8004} \]
\[\{ \text{lisp } (\text{cons } v_1 v_2, v_3, v_4, v_5, v_6, l) \ast \text{s } \ast \text{pc } (p + 328) \} \]

Here \(v_1 \ldots v_6\) are abstract s-expressions and lisp is a heap invariant.
LISP evaluation was defined as a tail-recursive function \textit{lisp\_eval} using only variables $v_1$…$v_6$, and operations for which the code generator has verified building blocks.

Compilation proceeds as normal and produces:

\[
\textit{lisp\_eval\_pre}(v_1, v_2, v_3, v_4, v_5, v_6, l) \Rightarrow \\
\{ \text{ lisp } (v_1, v_2, v_3, v_4, v_5, v_6, l) \ast s \ast \text{pc } p \} \\
p : \ldots \text{ the generated code } \ldots \\
\{ \text{ lisp } (\textit{lisp\_eval}(v_1, v_2, v_3, v_4, v_5, v_6, l)) \ast s \ast \text{pc } (p + 3012) \}
\]

This case study has evolved from the one reported in the proceeding. Ask, and I’ll tell more about the status of this project.
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2. how do extensions work? LISP interpreter.
3. design decisions and related work
Design decisions and related work

Why not verify the compiler? ¹

Why not instrument the code generation to produce proofs? ²,³

Does the compiler use heuristics to find the proofs? ⁴

¹ Leroy, POPL 2006; ² Pnueli, TACAS 1998; ³ Rinard, CC 1999; ⁴ Necula, PLDI 2000
Design decisions and related work

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Other questions?

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