Proof-producing decompilation and compilation

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Introduction

This talk concerns verification of functional correctness of machine code for commercial processors (ARM, PowerPC, x86 ...).

Outline of talk:

- motivation for decompilation into logic
- implementing decompilation
- compilation
Verification

Current approaches for machine-code verification:

- direct reasoning about *next*-state function.
- annotating code with assertions:

```assembly
xor eax, eax  
L1: test esi, esi
    jz L2  
    inc eax
    mov esi, [esi]
    jmp L1
L2:
```
Verification

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- direct reasoning about \textit{next}-state function.
- annotating code with assertions:

\begin{verbatim}
{...}
xor eax, eax
{...}
L1: test esi, esi
   jz L2
   {...}
   inc eax
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   jmp L1
L2: {...}
\end{verbatim}
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L2: {...}
```
```assembly
{...}
mov r0, #0
{...}
L: cmp r1, #0
    ldrne r1, [r1]
    addne r0, r0, #1
    bne L
```
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- Direct reasoning about next-state function.
- Annotating code with assertions:

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L1: test esi, esi
   jz L2
   {...}
   inc eax
   mov esi, [esi]
   jmp L1
L2: {...}
{???}

{???}
mov r0, #0
{???}

L: cmp r1, #0
   ldrne r1, [r1]
   addne r0, r0, #1
   bne L
   {???}
Proof reuse?
```
Our approach

Decompilation produces the following tail-recursive functions describing the effect of the code, $f$ for x86 and $f'$ for ARM:

\[
\begin{align*}
  f(eax, esi, m) &= \\
  &\text{let } eax = eax \otimes eax \text{ in} \\
  &\quad g(eax, esi, m) \\
  g(eax, esi, m) &= \\
  &\text{if } esi \& esi = 0 \\
  &\quad \text{then } (eax, esi, m) \text{ else} \\
  &\quad \text{let } eax = eax + 1 \text{ in} \\
  &\quad \text{let } esi = m(esi) \text{ in} \\
  &\quad g(eax, esi, m)
\end{align*}
\]
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Decompilation produces the following tail-recursive functions describing the effect of the code, $f$ for x86 and $f'$ for ARM:

$$f(eax, esi, m) = \begin{cases} \text{let } eax = eax \otimes eax \text{ in } g(eax, esi, m) \\ g(eax, esi, m) = \begin{cases} (eax, esi, m) \text{ if } esi \& esi = 0 \\ \text{let } eax = eax+1 \text{ in} \\ \text{let } esi = m(esi) \text{ in} \\ g(eax, esi, m) \end{cases} \end{cases}$$

$$f'(r_0, r_1, m) = \begin{cases} \text{let } r_0 = 0 \text{ in } g'(r_0, r_1, m) \\ g'(r_0, r_1, m) = \begin{cases} (r_0, r_1, m) \text{ if } r_1 = 0 \\ \text{let } r_1 = m(r_1) \text{ in} \\ \text{let } r_0 = r_0+1 \text{ in} \\ g'(r_0, r_1, m) \end{cases} \end{cases}$$

Advantages:
1. no need for knowledge of the next-state function;
2. suitable for proofs in HOL, and
3. proof reuse,

$f = f'$ using $w \& w = w$ and $w \otimes w = 0$.
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Decomposition produces the following tail-recursive functions describing the effect of the code, \( f \) for x86 and \( f' \) for ARM:

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\begin{align*}
\text{f}(\text{eax}, \text{esi}, m) &= \\
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\quad \text{g}(\text{eax}, \text{esi}, m)
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g(\text{eax}, \text{esi}, m) &= \\
\quad \text{if } \text{esi} \& \text{esi} = 0 \\
\quad \text{then } (\text{eax}, \text{esi}, m) \text{ else} \\
\quad \quad \text{let } \text{eax} = \text{eax}+1 \text{ in} \\
\quad \quad \text{let } \text{esi} = m(\text{esi}) \text{ in} \\
\quad \quad \text{g}(\text{eax}, \text{esi}, m)
\end{align*}
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\[
\begin{align*}
\text{f}'(r_0, r_1, m) &= \\
\quad \text{let } r_0 = 0 \text{ in} \\
\quad \text{g}'(r_0, r_1, m)
\\
g'(r_0, r_1, m) &= \\
\quad \text{if } r_1 = 0 \\
\quad \text{then } (r_0, r_1, m) \text{ else} \\
\quad \quad \text{let } r_1 = m(r_1) \text{ in} \\
\quad \quad \text{let } r_0 = r_0+1 \text{ in} \\
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Produced theorem

How does $f$ relate to the x86 code?

Answer: For each run, the decompiler automatically:

1. generates a function $f$, and
2. proves a theorem relating the function to the code:
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2. proves a theorem relating the function to the code:

$$\{ (eax, esi, m) \text{ is } (eax, esi, m) \ast eip \ p \ast f_{pre}(eax, esi, m) \}$$

$$p : 31C085F67405408B36EBF7$$

$$\{ (eax, esi, m) \text{ is } f(eax, esi, m) \ast eip \ (p+11) \}$$

Here $eax$, $esi$, $m$ and $eip$ (program counter) assert values of resources and ‘$(x, y, z) \text{ is } (a, b, c)$’ abbreviates $(x \ a) \ast (y \ b) \ast (z \ c)$. 
Verification

The decompiler automates machine specific proofs and leaves the user (verifier) to prove properties of the generated function $f$.

Suppose we have proved

\[ \forall xs \ w \ a \ m. \ list(xs, a, m) \Rightarrow f(w, a, m) = (\text{length}(xs), 0, m) \]
\[ \forall xs \ w \ a \ m. \ list(xs, a, m) \Rightarrow f_{\text{pre}}(w, a, m) \]

for an appropriate definition of $list$. 
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Then we have:

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\text{list}(xs, esi, m) \Rightarrow
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for an appropriate definition of $list$.

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$$list(xs, esi, m) \Rightarrow$$

$$\{ (eax, esi, m) \text{ is } (eax, esi, m) * \text{eip} ~ p * f_{\text{pre}}(eax, esi, m) \}$$

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Suppose we have proved

\[ \forall xs \ w \ a \ m. \ list(xs, a, m) \implies f(w, a, m) = (\text{length}(xs), 0, m) \]
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Then we have:

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Outline of talk:

- motivation for decompilation into logic
- implementing decompilation
- compilation
Algorithm

Given some code, the decompiler:

1. derives a specification for each instruction;
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3. for each code segment:
   a) derives a specification for one pass through the code;
   b) generates a function describing effect of code;
   c) for loops, instantiates special loop rule.
Algorithm

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1. derives a specification for each instruction;
2. discovers the control flow;
3. for each code segment:
   a) derives a specification for one pass through the code;
   b) generates a function describing effect of code;
   c) for loops, instantiates special loop rule.
4. composes the top-level specifications and repeats step 3 until all of the code is described by one specification.
Proving loops

**Approach:** assume existence of termination proof, use induction from termination proof to prove loop.
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The decompiler models loops as tail-recursive functions $k$, with appropriate instantiations of $F$, $G$ and $D$, where:

$$k(x) = \text{if } G(x) \text{ then } k(F(x)) \text{ else } D(x)$$
**Proving loops**

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$$k(x) = \text{if } G(x) \text{ then } k(F(x)) \text{ else } D(x)$$

We define $\text{pre}(x)$ to state that there exists and invariant which guarantees that $k$ terminates when applied to $x$. Here $\prec$ is some well-founded relation.

$$\text{pre}(x) = \exists \text{inv. } \text{inv}(x) \land \exists \prec. \forall y. \text{inv}(y) \land G(y) \Rightarrow \text{inv}(F(y)) \land F(y) \prec y$$
The loop rule, used by the decompiler, for function

\[ k(x) = \text{if } G(x) \text{ then } k(F(x)) \text{ else } D(x) \]

is the following: for any resource assertions \( \text{res} \) and \( \text{res}' \),

\[
(\forall x. \ G(x) \Rightarrow \{\text{res } x\} c \{\text{res } F(x)\}) \land \\
(\forall x. \neg G(x) \Rightarrow \{\text{res } x\} c \{\text{res}' D(x)\}) \\
\Rightarrow (\forall x. \{\text{res } x \ast \text{pre}(x)\} c \{\text{res}' k(x)\})
\]
The loop rule, used by the decompiler, for function

\[ k(x) = \begin{cases} G(x) \text{ then } k(F(x)) & \text{else } D(x) \end{cases} \]

is the following: for any resource assertions \( \text{res} \) and \( \text{res}' \),

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\Rightarrow (\forall x. \{\text{res } x \ast \text{pre}(x)\} c \{\text{res}' k(x)\})
\]

In our x86 example the loop uses assertions:

\[
\begin{align*}
\text{res } x &= (\text{eax, esi, m}) \text{ is } x \ast \text{eip } p \\
\text{res}' x &= (\text{eax, esi, m}) \text{ is } x \ast \text{eip } (p+9)
\end{align*}
\]
The loop rule is derived from the following induction, provable from the definition of \(pre\) and well-founded relation:

\[
\forall \varphi. \ (\forall x. G(x) \land \varphi(F(x)) \Rightarrow \varphi(x)) \land \\
(\forall x. \neg G(x) \Rightarrow \varphi(x)) \\
\Rightarrow (\forall x. pre(x) \Rightarrow \varphi(x))
\]

The proof of the loop rule uses the following composition:

\[
\{\text{res } x\} \ c \ \{\text{res } F(x)\} \land \{\text{res } F(x)\} \ c \ \{\text{res' } k(x)\} \\
\Rightarrow \ \{\text{res } x\} \ c \ \{\text{res' } k(x)\} \\
\Rightarrow \ \{\text{res } x\} \ c \ \{\text{res' } k(x)\}
\]
Decompilation

For most part proof-producing decompilation is just:
1. deriving specifications for individual instructions;
2. composing them; and
3. instantiating a loop rule.

Failure prone heuristic are largely avoided.

This method does not support advanced control-flow, e.g. computed jumps and code pointers.

However, it does support non-nested loops, procedure calls and, in an awkward way, procedural recursion.
Outline of talk:

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- implementing decompilation
- compilation
Proof-producing compilation

For a simple compiler, given a function $f$:

- generate code;
- run decompiler to get $f'$;
- automatically prove $f = f'$.

Works well, even for functions $f$ which are hundreds of lines long.
Proof-producing compilation

For a simple compiler, given a function $f$:

- generate code;
- run decompiler to get $f'$;
- automatically prove $f = f'$.

Works well, even for functions $f$ which are hundreds of lines long.

Each expression in $f$ must implementable in the target language, e.g. “let $eax = eax + 1$ in” and “if $eax < 400$ then ... else ...”

However, we can do better...
Proof-producing compilation (continued)

Suppose we have a specification for allocation on a garbage collected heap $h$, which allows allocation of a new element if the size of the current heap $h$ does not exceed the limit $l$.

$$\{ \text{heap} (v_1, v_2, v_3, v_4, h, l) \ast \text{eip } p \ast \text{size}(h) < l \}$$

...code...

$$\{ \text{heap} (\text{fresh}(h), v_2, v_3, v_4, h[\text{fresh}(h) \mapsto (v_1, v_2)], l) \ast \text{eip } (p+416) \}$$
Proof-producing compilation (continued)

Suppose we have a specification for allocation on a garbage collected heap $h$, which allows allocation of a new element if the size of the current heap $h$ does not exceed the limit $l$.

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...code...

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Such specifications can be fed into the automation so that the compiler can handle:

"let $(v_1, h) = (\text{fresh}(h), h[\text{fresh}(h) \mapsto (v_1, v_2)])$ in"

The side-condition $\text{size}(h) < l$ is recorded in the precondition of the theorem from the decompiler.
Conclusions

Decompile and compilation are based on:
(a) modelling loops as tail-recursion, and
(b) proving (a) correct using termination proofs.

Details described in paper available at: www.cl.cam.ac.uk/mom22

Acknowledgments: I would like to thank Mike Gordon, Konrad Slind, Thomas Tuerk, Anthony Fox, Susmit Sarkar, Peter Sewell, Boris Feigin, Max Bolingbroke, John Regehr, Lu Zhao and Matthew Parkinson for discussions.
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Decompilation and compilation are based on:

(a) modelling loops as tail-recursion, and
(b) proving (a) correct using termination proofs.

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My questions for you:

► How has the correspondence between loops and tail-recursion been formally proved before?
► Have termination proofs been used for this?

Questions?

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