Tick exercise 2

MPhil ACS & Part III course,
Functional Programming: Implementation, Specication and Verification,
Michaelmas term, 2013.

Deadline: **4pm, 21 Nov 2013**
Assessment: marked pass or fail, this exercise is 10 % of the final course mark
Return solutions to: Kate Cisek, FS05

The exercise:

1. Consider the ML expression: `fn i => i + 1`
   (a) Explain how this expression is represented in the intermediate language (IL) from Lectures 8 and 10, using the IL's deBruijn index notation.
   (b) The ML expression above evaluates in the semantics to a closure value. How is this closure value represented in the bytecode semantics from Lectures 8 and 10? [Hint: each bytecode value is either an integer (`Int`), a block containing a list of values (`Blk`), or a code pointer (`Ptr`)]
   (c) Sketch the bytecode that implement the ML expression above. [Hint: the bytecode ought to consist of two parts: one for the body of the `fn`-expression, *i.e.* `i + 1`, and another that constructs the bytecode value representing the closure, *i.e.* constructs your answer to part 1(b).]

2. Consider type inference applied to the following ML expression.

   `if f 10 then g k else 4`

   Derive the typing constraints that the algorithm from Lecture 9 produces. In other words: using the rules from slides 22–25, find a constraint set c such that: [Clarification added on 13 Nov 2013: consider $\Gamma = \texttt{f} : t, \texttt{g} : u, \texttt{k} : v$.]

   $$\Gamma \ \vdash \texttt{(if f 10 then g k else 4)} : \mathsf{int} \ \Downarrow \ c$$

3. Lecture 6 sketches, on slides 48–56, the proof of a machine-code Hoare triple that describes, in terms of `HEAP`, the `load r2,[r1]` instruction as an assignment of `car x1` to variable `x2`. Using the definition of heap_inv, sexp_range and sexp_repr, sketch a proof of the following key lemma:

   $$\forall x_0\ x\ y\ x_2\ x_3\ m\ r_0\ r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8.$$
   $$\mathsf{heap\_inv}\ (x_0, \mathsf{Dot}\ x\ y, x_2, x_3)\ (m, r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8) \implies$$
   $$\mathsf{heap\_inv}\ (x_0, \mathsf{Dot}\ x\ y, x, x_3)\ (m, r_0, r_1, m(r_1), r_3, r_4, r_5, r_6, r_7, r_8)$$