Memory abstraction, verification of a garbage collector

Lecture 5

MPhil ACS & Part III course, Functional Programming: Implementation, Specification and Verification

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Memory

Machine code:

- load, store instructions access memory
- memory is large flat array

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- assertion about a region of memory:

```
MEM m = λs. domain s = { Mem a | a ∈ domain m} ∧
            ∀a ∈ domain m. s (Mem a) = Word (m a)
```

single-word memcopy

Using MEM we can decompile

```
“ load r0,[r2]
  store r0,[r1] ”
```

The extracted function:

```
memcopy (r1,r2,m) =
  let r0 = m r2 in
  let m = m[r1 ↦ r0] in
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The certificate theorem:

```
memcopy_pre (r1,r2,m) ⟹
{ PC pc * R 0 r0 * R 1 r1 * R 2 r2 * MEM m }
  “ load r0,[r2]
    store r0,[r1] ”
{ let (r0,r1,r2,m) = memcopy (r1,r2,m) in
    PC (pc + 8) * R 0 r0 * R 1 r1 * R 2 r2 * MEM m }
```

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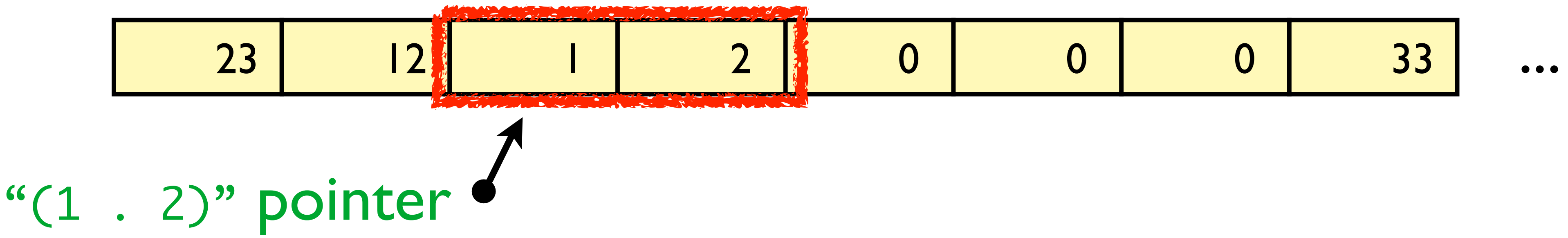
Representation in memory:



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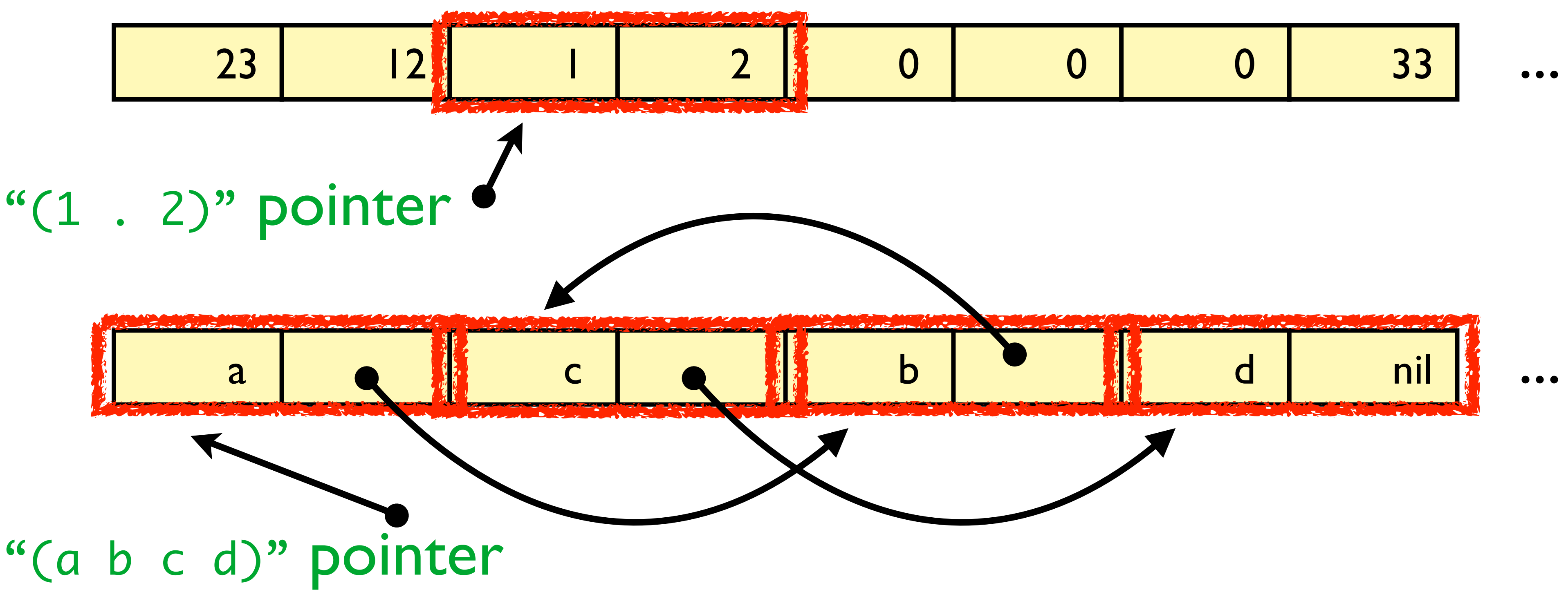

Garbage collection

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Garbage collection (GC) finds and deletes unused data.

GC is invisible to user (part of memory abstraction).

Garbage collection (cont.)

Jargon:

- moving or non-moving?
- copying or mark-and-sweep or mark-and-don't-sweep?
- generational or not?
- stop-the-world or incremental or concurrent?
- precise or conservative?

Garbage collection (cont.)

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This lecture: verification of a simple copying GC.

Organising a verification proof

Task: construction of verified code for GC routine

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Step 5: prove connection between impl. of Step 3 and 4.

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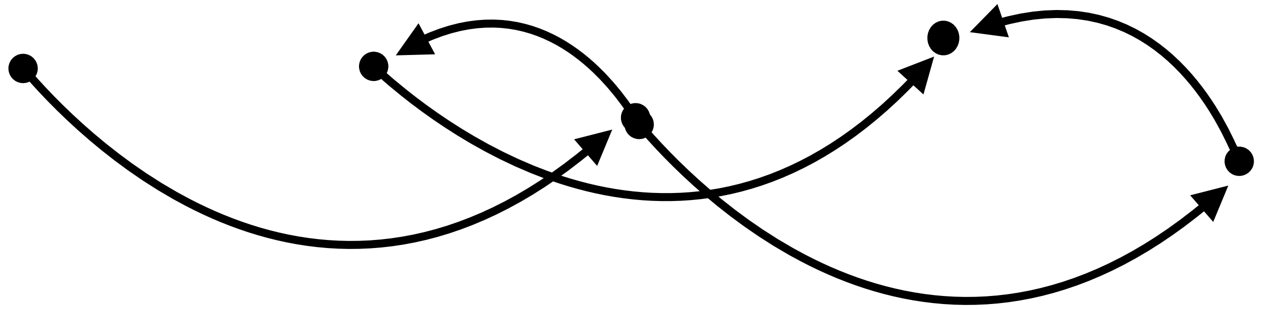

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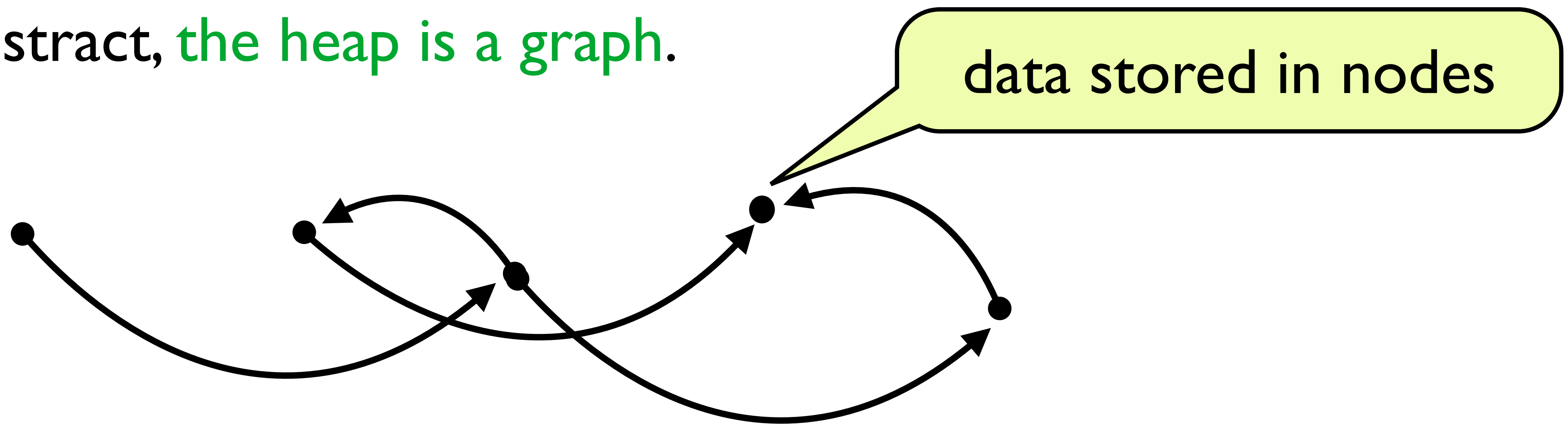

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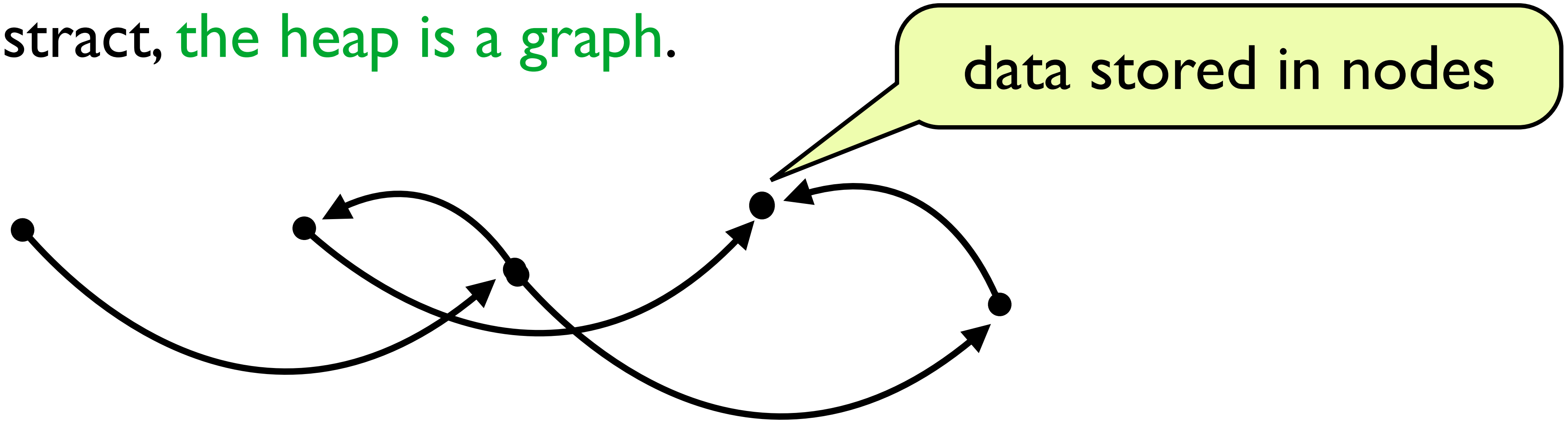

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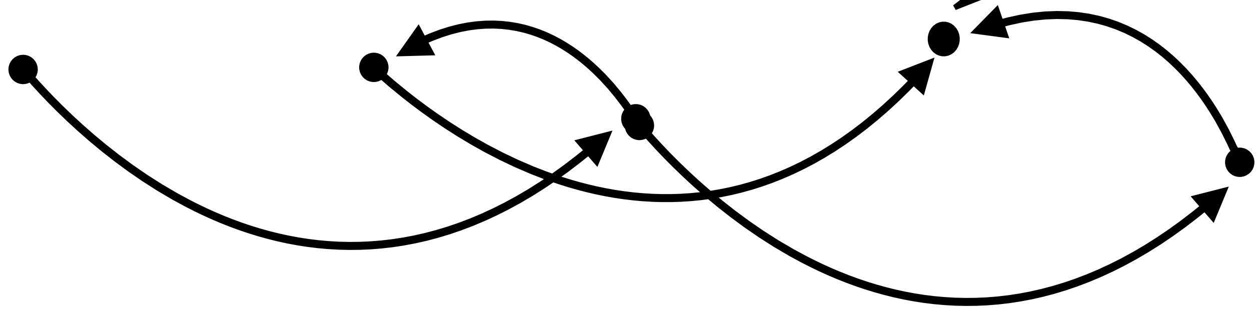

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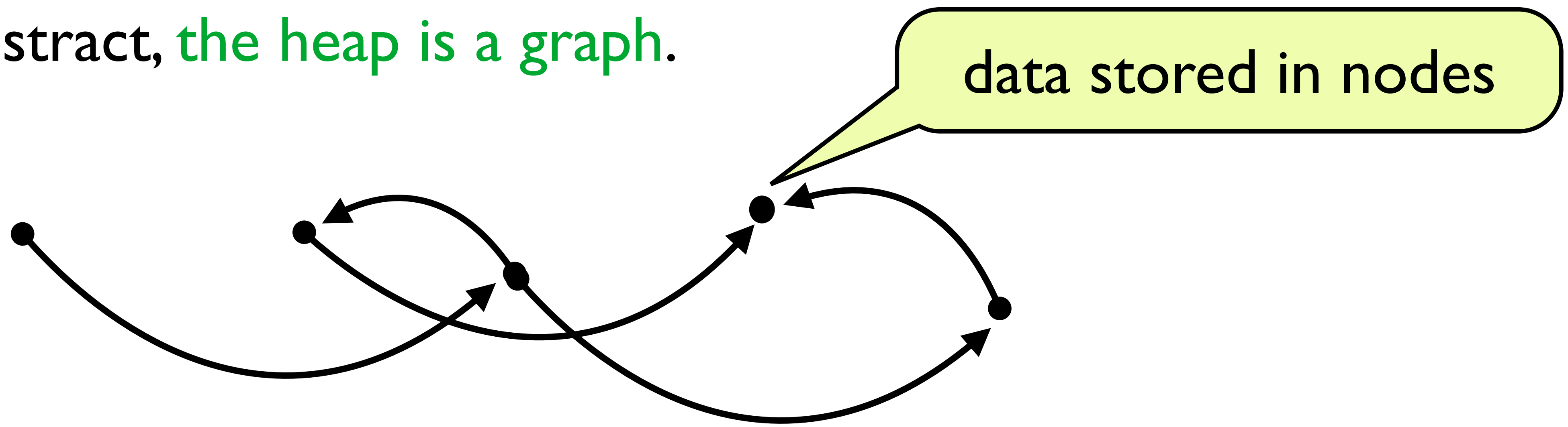

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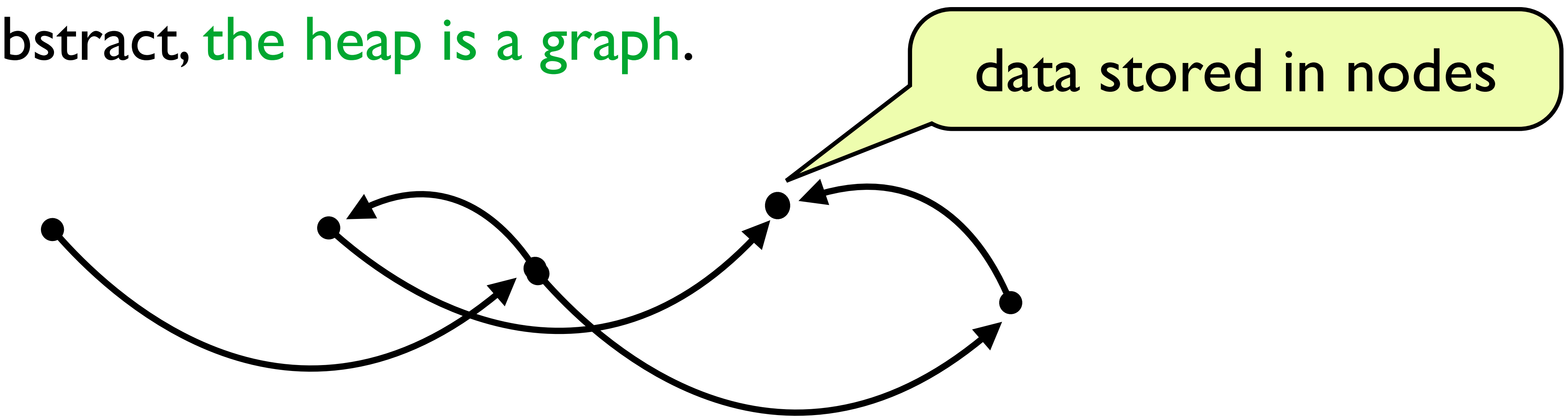

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at this level, type variable

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Reachability

GC must not delete reachable nodes. Reachable:

$$\frac{a \in \mathsf{set}\ roots}{a \in \mathsf{reach}\ (h, roots)}$$

$$\frac{a \in \mathsf{set}\ as \ \wedge\ h(b) = (as, data)\ \wedge\ b \in \mathsf{reach}\ (h, roots)}{a \in \mathsf{reach}\ (h, roots)}$$

A full GC ought to only keep reachable nodes:

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restricts domain of h function to reach set

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We are allowed to apply a renaming function.

$$\textsf{domain}\ (\textsf{rename}\ f\ \ h) = \textsf{image}\ f\ \ (\textsf{domain}\ h)$$
$$(\textsf{rename}\ f\ \ h)(f(x)) = (\textsf{map}\ f\ \ as, d) \qquad \text{whenever } h(x) = (as, d)$$

Define:

$$\frac{f \circ f = \textsf{id}}{(h, roots) \xrightarrow{\textsf{translate}} (\textsf{rename}\ f\ \ h, \textsf{map}\ f\ \ roots)}$$

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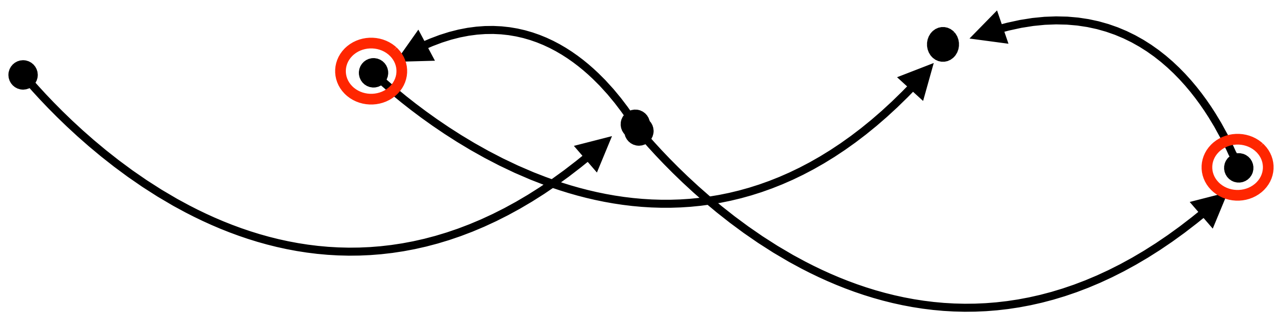
The specification of a full moving GC:

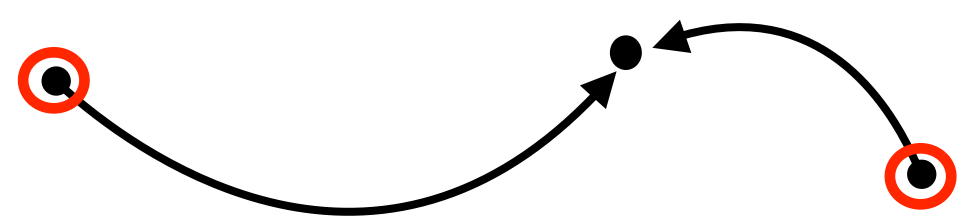
$$x \xrightarrow{\mathsf{gc}} y \quad = \quad (\mathsf{filter}\ x) \xrightarrow{\mathsf{translate}} y$$

Example

Initial heap graph (with roots marked red):



After GC:



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$$\frac{a \in z \;\wedge\; f(a) \neq a}{(h, x, \ldots}$$

finalise heap cell

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Correctness

Correctness theorem:

$$\forall h\ h_2\ roots\ x\ f.$$
$$(h, \{\}, \{\}, \mathsf{set}\ roots, \mathsf{id}) \xrightarrow{\mathsf{step}}{}^* (h_2, x, \{\}, \{\}, f) \ \wedge\ \mathsf{ok\_heap}\ (h, roots) \implies$$
$$(h, roots) \xrightarrow{\mathsf{gc}} (h_2 | x, \mathsf{map}\ f\ roots)$$

$$\text{where}\quad \mathsf{ok\_heap}\ (h, roots) \ = \ \mathsf{pointers}\ h \cup \mathsf{set}\ roots \subseteq \mathsf{domain}\ h$$
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Proof: we prove that an invariant is maintained

$$\forall x\ s\ t.\ \ \mathsf{inv}\ x\ s\ \wedge\ s \xrightarrow{\mathsf{step}} t \implies \mathsf{inv}\ x\ t$$

and sufficient. Invariant on next slide...

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terminates when nothing left to-do

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$$(h, \{\}, \{\}, \mathsf{set}\ roots, \mathsf{id}) \xrightarrow{\mathsf{step}}{}^{*}\ (h_2, x, \{\}, \{\}, f)\ \wedge\ \mathsf{ok\_heap}\ (h, roots)\ \Longrightarrow$$
$$(h, roots) \xrightarrow{\mathsf{gc}} (h_2 \lfloor x, \mathsf{map}\ f\ roots)$$

$$\text{where}\quad \mathsf{ok\_heap}\ (h, roots)\ =\ \mathsf{pointers}\ h \cup \mathsf{set}\ roots \subseteq \mathsf{domain}\ h$$
$$\mathsf{pointers}\ h\ =\ \{\ x \mid \exists a\ as\ d.\ x \in \mathsf{set}\ as \wedge h(a) = (as, d)\ \}$$

Proof: we prove that an invariant is maintained

$$\forall x\ s\ t.\ \ \mathsf{inv}\ x\ s\ \wedge\ s \xrightarrow{\mathsf{step}} t\ \Longrightarrow\ \mathsf{inv}\ x\ t$$

and sufficient. Invariant on next slide...

Correctness

starts off with roots to-be moved

Correctness theorem:

terminates when nothing left to-do

$$\forall h\ h_2\ roots\ x\ f.$$
$$(h, \{\}, \{\}, \textsf{set}\ roots, \textsf{id}) \xrightarrow{\textsf{step}}{}^{*} (h_2, x, \{\}, \{\}, f)\ \wedge\ \textsf{ok\_heap}\ (h, roots) \implies$$
$$(h, roots) \xrightarrow{\textsf{gc}} (h_2{\downharpoonright}x, \textsf{map}\ f\ roots)$$

$$\begin{aligned} \text{where}\quad \textsf{ok\_heap}\ (h, roots) &= \textsf{pointers}\ h \cup \textsf{set}\ roots \subseteq \textsf{domain}\ h \\ \textsf{pointers}\ h &= \{\ x \mid \exists a\ as\ d.\ x \in \textsf{set}\ as \wedge h(a) = (as, d)\ \} \end{aligned}$$

any such terminating execution is a valid GC execution

Proof: we prove that an invariant is maintained

$$\forall x\ s\ t.\ \ \textsf{inv}\ x\ s\ \wedge\ s \xrightarrow{\textsf{step}} t \implies \textsf{inv}\ x\ t$$

and sufficient. Invariant on next slide...

Invariant

The lengthy invariant:

$$
\begin{array}{rl}
 & \textsf{inv}\ (h_0, roots)\ (h, x, y, z, f) = \\
0 & \quad \textsf{let}\ old = (\textsf{domain}\ h \cup \{\ a \mid f(a) \neq a\ \}) - (x \cup y)\ \textsf{in} \\
1 & \qquad (x \cap y = \{\}) \wedge (f \circ f = \textsf{id}) \wedge \\
2 & \qquad \textsf{pointers}\ (h\lfloor x) \subseteq x \cup y\ \wedge \\
3 & \qquad \textsf{pointers}\ (h\lfloor x^c) \subseteq old\ \wedge \\
4 & \qquad \textsf{pointers}\ (h\lfloor y) \cup \textsf{set}\ roots \subseteq \textsf{image}\ f\ (x \cup y) \cup z \subseteq \textsf{reach}\ (h_0, roots)\ \wedge \\
5 & \qquad (\forall a.\ a \in z \implies \textsf{if}\ f(a) = a\ \textsf{then}\ a \in old\ \textsf{else}\ f(a) \in x \cup y) \wedge \\
6 & \qquad (\forall a.\ f(a) \neq a \implies \neg(a \in x \cup y \iff f(a) \in x \cup y)) \wedge \\
7 & \qquad (\forall a.\ a \in x \cup y \iff f(a) \neq a \wedge a \in \textsf{domain}\ h) \wedge \\
8 & \qquad \textsf{domain}\ h = \textsf{image}\ f\ (\textsf{domain}\ h_0) \wedge \\
9 & \qquad (\forall a\ as\ d.\ f(a) \in \textsf{domain}\ h \wedge h(f(a)) = (as, d) \implies \\
 & \qquad\qquad\qquad h_0(a) = \textsf{if}\ f(a) \in x\ \textsf{then}\ (\textsf{map}\ f\ as, d)\ \textsf{else}\ (as, d))
\end{array}
$$

Most effort is spent finding the invariant.

First-order prover can automate much of the proof.

Implementation with memory

Next refinement introduces an abstract memory.

Memory consists of

$$
\begin{array}{lll}
\text{Block } (as, l, d) & \text{---} & \text{block of data, e.g. a cons-cell} \\
\text{Ref } a & \text{---} & \text{record of where data has moved} \\
\text{Emp} & \text{---} & \text{empty or `don't care'}
\end{array}
$$

Relation to small-step relation's state:

$$
\begin{array}{ll}
m(a) = \text{Block } (h(a)) & \text{if } a \in \text{domain } h \\
m(a) = \text{Ref } (f(a)) & \text{if } a \notin \text{domain } h \text{ and } f(a) \neq a \\
m(a) = \text{Emp} & \text{if } a \notin \text{domain } h \text{ and } f(a) = a
\end{array}
$$

Implementation with memory

move (RHS n, j, m) $=$ (RHS n, j, m)
move (LHS a, j, m) $=$ case $m(a)$ of
  Ref $i \rightarrow$ (LHS i, j, m)
  $|$ Block $(as, l, d) \rightarrow$
    let $m = m[a \mapsto$ Ref $j]$ in
    let $m = m[j \mapsto$ Block $(as, l, d)]$ in
      (LHS $j, j + l + 1, m$)

move_list $([], j, m) = ([], j, m)$
move_list $(r{::}rs, j, m) =$
  let $(r, j, m) =$ move (r, j, m) in
  let $(rs, j, m) =$ move_list (rs, j, m) in
    $(r{::}rs, j, m)$

readBlock (Block x) $= x$
cut $(i, j)\ m\ =\ \lambda k.$ if $i \leq k \wedge k < j$ then $m\,k$ else Emp

loop $(i, j, m)\ =$
  if $i = j$ then (i, m) else
    let $(as, l, d) =$ readBlock $(m\ i)$ in
    let $(as, j, m) =$ move_list (as, j, m) in
    let $m = m[i \mapsto$ Block $(as, l, d)]$ in
      loop $(i + l + 1, j, m)$

collector $(roots, b, i, e, b_2, e_2, m)\ =$
  let $(b_2, e_2, b, e) = (b, e, b_2, e_2)$ in
  let $(roots, j, m) =$ move_list $(roots, b, m)$ in
  let $(i, m) =$ loop (b, j, m) in
  let $m =$ cut $(b, i)\ m$ in
    $(roots, b, i, e, b_2, e_2, m)$

Correctness

Correctness theorem:

$$
\begin{array}{l}
\forall h\ roots\ roots_2\ x\ y. \\
\quad \textsf{ok\_mem\_heap}\ (h, roots)\ x\ \wedge\ \textsf{collector}\ (roots, x) = (roots_2, y)\ \Longrightarrow \\
\quad \exists h_2.\ \textsf{ok\_mem\_heap}\ (h_2, roots_2)\ y\ \wedge\ (h, roots) \xrightarrow{\textsf{gc}} (h_2, roots_2)
\end{array}
$$

Correctness

if abstract state is correctly represented, then ...

Correctness theorem:

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for any execution of implementation...

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some new abstract heap exists such that ...

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Correctness theorem

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some new abstract heap exists such that ...

specification is met

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\end{array}
$$

some new abstract heap exists such that ...

specification is met

Proof: again uses a lengthy invariant

$$
\begin{array}{l}
\textsf{mem\_inv}\ (h_0, roots_0, h, f)\ (b, i, j, e, b_2, e_2, m, z) =\\
\quad b \le i \le j \le e \wedge (e < b_2 \vee e_2 < b)\ \wedge\\
\quad (\forall a.\ a \notin b_2 ... e_2 \cup b ... j\ \Longrightarrow\ m(a) = \textsf{Emp})\ \wedge\\
\quad \textsf{part\_heap}\ (b, i)\ m\ (i - b) \wedge \textsf{part\_heap}\ (i, j)\ m\ (j - i)\ \wedge\\
\quad (\exists k.\ \textsf{part\_heap}\ (b_2, e_2)\ m\ k \wedge k \le e - j)\ \wedge\\
\quad \textsf{ref\_mem}\ (h, f)\ m \wedge \textsf{ok\_heap}\ (h_0, roots_0)\ \wedge\\
\quad (h_0, \{\}, \{\}, \textsf{set}\ roots_0, \textsf{id}) \xrightarrow{\textsf{step}}{*}\ (h, \textsf{domain}\ h \cap (b ... i), \textsf{domain}\ h \cap (i ... j), z, f)
\end{array}
$$

Correctness

if abstract state is correctly represented, then ...

Correctness theorem

for any execution of implementation...

$\forall h\ roots\ roots_2\ x\ y.$

$\quad \mathsf{ok\_mem\_heap}\ (h, roots)\ x\ \wedge\ \mathsf{collector}\ (roots, x) = (roots_2, y)\ \Longrightarrow$

$\quad \exists h_2.\ \mathsf{ok\_mem\_heap}\ (h_2, roots_2)\ y\ \wedge\ (h, roots) \xrightarrow{\mathsf{gc}} (h_2, roots_2)$

some new abstract heap exists such that ...

specification is met

Proof: again uses a lengthy invariant

$\mathsf{mem\_inv}\ (h_0, roots_0, h, f)\ (b, i, j, e, b_2, e_2, m, z) =$

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$\quad (\forall a.\ a \notin b_2 ... e_2 \cup b ... j\ \Longrightarrow\ m(a) = \mathsf{Emp})\ \wedge$

$\quad \mathsf{part\_heap}\ (b, i)\ m\ (i - b) \wedge \mathsf{part\_heap}\ (i, j)\ m$

$\quad (\exists k.\ \mathsf{part\_heap}\ (b_2, e_2)\ m\ k \wedge k \le e - j)\ \wedge$

$\quad \mathsf{ref\_mem}\ (h, f)\ m \wedge \mathsf{ok\_heap}\ (h_0, roots_0)\ \wedge$

$\quad (h_0, \{\}, \{\}, \mathsf{set}\ roots_0, \mathsf{id}) \xrightarrow{\mathsf{step}} {*}\ (h, \mathsf{domain}\ h \cap (b...i), \mathsf{domain}\ h \cap (i...j), z, f)$

most is trivial book-keeping
(where/how state is repr.)

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$$\mathsf{ok\_mem\_heap}\ (h, roots)\ x\ \wedge\ \mathsf{collector}\ (roots, x) = (roots_2, y)\ \Longrightarrow$$
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$$\mathsf{part\_heap}\ (b, i)\ m\ (i - b) \wedge \mathsf{part\_heap}\ (i, j)\ m$$
$$(\exists k.\ \mathsf{part\_heap}\ (b_2, e_2)\ m\ k \wedge k \le e - j)\ \wedge$$
$$\mathsf{ref\_mem}\ (h, f)\ m \wedge \mathsf{ok\_heap}\ (h_0, roots_0)\ \wedge$$
$$(h_0, \{\}, \{\}, \mathsf{set}\ roots_0, \mathsf{id}) \xrightarrow{\mathsf{step}}{*}\ (h, \mathsf{domain}\ h \cap (b...i), \mathsf{domain}\ h \cap (i...j), z, f)$$

most is trivial book-keeping
(where/how state is repr.)

reason for correctness inherited

Some assembly code

ARM

```
    tst r2, #3
    bne L0
    ldr r4, [r2]
    tst r4, #3
    streq r4, [r1]
    beq L0
    str r3, [r1]
    str r4, [r3]
    str r3, [r2], #4
    mov r4, r4, LSR #10
    add r3, r3, #4
L1: cmp r4, #0
    beq L0
    ldr r5, [r2]
    sub r4, r4, #1
    add r2, r2, #4
    str r5, [r3]
    add r3, r3, #4
    b L1
L0:
```

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    add r3, r3, #4
    b L1
L0:
```

Decompilation produces functions, e.g.

$$
\begin{array}{l}
\text{mc\_move\_loop}\ (r_2, r_3, r_4, g) = \\
\quad \text{if } r_4 = 0 \text{ then } (r_2, r_3, r_4, g) \text{ else} \\
\quad\quad \text{let } r_5 = g(r_2) \text{ in} \\
\quad\quad \text{let } r_4 = r_4 - 1 \text{ in} \\
\quad\quad \text{let } r_2 = r_2 + 4 \text{ in} \\
\quad\quad \text{let } g = g[r_3 \mapsto r_5] \text{ in} \\
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\end{array}
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```

Decompilation produces functions, e.g.

mc_move_loop $(r_2, r_3, r_4, g) =$
  if $r_4 = 0$ then (r_2, r_3, r_4, g) else
    let $r_5 = g(r_2)$ in
    let $r_4 = r_4 - 1$ in
    let $r_2 = r_2 + 4$ in
    let $g = g[r_3 \mapsto r_5]$ in
    let $r_3 = r_3 + 4$ in
      mc_move_loop (r_2, r_3, r_4, g)

Some assembly code

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Carefully written code for other architectures (x86, PowerPC etc.) decompiles to the same function in logic.

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```

Carefully written code for other architectures (x86, PowerPC etc.) decompiles to the same function in logic. Proof reuse!

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L0:
```

x86

```
    test ecx, 3
    jne L0
    mov ebx, [ecx]
    test ebx, 3
    jne L2
    mov [eax], ebx
    jmp L0
L2: mov [eax], edx
    mov [edx], ebx
    mov [ecx], edx
    shr ebx, 10
    add edx, 4
    add ecx, 4
L1: cmp ebx, 0
    je L0
    mov edi, [ecx]
    dec ebx
    add ecx, 4
    mov [edx], edi
    add edx, 4
    jmp L1
L0:
```

PowerPC

```
    andi. 0, 2, 3
    bne L0
    lwz 4, 0(2)
    andi. 0, 4, 3
    bne L2
    stw 4, 0(1)
    b L0
L2: stw 3, 0(1)
    stw 4, 0(3)
    stw 3, 0(2)
    srawi 4, 4, 10
    addi 3, 3, 4
    addi 2, 2, 4
L1: cmplwi 4,0
    beq L0
    lwz 5, 0(2)
    addi 4, 4, -1
    addi 2, 2, 4
    stw 5, 0(3)
    addi 3, 3, 4
    b L1
L0:
```

Proving final connection

Correctness theorem:

$$\forall x\ y\ z.\ \ \mathsf{ok\_mc\_heap}\ x\ y\ z \implies \mathsf{ok\_mc\_heap}\ x\ (\mathsf{collector}\ y)\ (\mathsf{mc\_collector}\ z)$$

Proving final connection

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relation between abstract memory and concrete memory

Proving final connection

abstract memory impl.

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relation between abstract memory and concrete memory

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decompiler extracted functions

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relation between abstract memory and concrete memory

relation maintained

The ok_mc_heap relation:

- specifies the exact layout in machine memory
- separation logic notation used for brevity
- lengthy definition omitted

Result: memory abstraction

A high-level theorem about the machine code for GC:

```
{ HEAP abs_state * PC pc }
  “ entire GC implementation with entry point pc ”
{ HEAP abs_state * PC (pc + length_of_gc_impl) }
```

where `HEAP abs_state = ∃m regs. MEM m * ... *`
`pure (high_low_rel abs_state m regs)`

GC implementation:

- always terminates
- maintains the memory abstraction
- is transparent: no visible change in high-level view of state
- even though all addresses renamed

Summary

Garbage collection

- reclaims unused memory
- automatic memory management
- part of memory abstraction
- copying collection moves data

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Attempt to verify complex implementation at low level of abstraction.

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- best split into separate layers of abstraction
- proof by (data-)refinement separates
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Common pitfall:
Attempt to verify complex implementation at low level of abstraction.

- best split into separate layers of abstraction
- proof by (data-)refinement separates
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Tip: powerful proof automation can be used
if problem is phrased suitably