Whole Theory Transferred to ST

Definitions

$$\begin{array}{l}
\vdash \texttt{AbsGroupAxioms}\ G\ \bullet\ \sim\ \imath\ \texttt{=}\\
\quad (\bullet\ \in\ G \rightarrow G \rightarrow G)\ \wedge\\
\quad (\sim\ \in\ G \rightarrow G)\ \wedge\\
\quad (\imath\ \in\ G)\ \wedge\\
\quad \mathsf{Assoc}\ G\ \bullet\ \wedge\\
\quad \mathsf{Inv}\ G\ \bullet\ \sim\ \imath\ \wedge\\
\quad \mathsf{Id}\ G\ \bullet\ \imath
\end{array}$$

$$\begin{array}{l}
\vdash \mathsf{Assoc}\ G\ \bullet\ \texttt{=}\\
\quad \forall \texttt{x y z} \in G.\\
\qquad \texttt{x} \bullet (\texttt{y} \bullet \texttt{z}) = (\texttt{x} \bullet \texttt{y}) \bullet \texttt{z}
\end{array}$$

$$\vdash \mathsf{Inv}\ G\ \bullet\ \sim\ \imath\ = \forall \texttt{x} \in G.\ \texttt{x} \bullet \sim\texttt{x} = \imath$$

$$\vdash \mathsf{Id}\ G\ \bullet\ \imath\ = \forall \texttt{x} \in G.\ \texttt{x} \bullet \imath = \texttt{x}$$

Theorems

$$\begin{array}{ll}
\mathit{Lemma1} & \texttt{AbsGroupAxioms}\ G\ \bullet\ \sim\ \imath\\
 & \vdash\ \forall \texttt{x} \in G.\ \sim\texttt{x} \bullet \texttt{x} = \imath
\end{array}$$

$$\begin{array}{ll}
\mathit{Lemma2} & \texttt{AbsGroupAxioms}\ G\ \bullet\ \sim\ \imath\\
 & \vdash\ \forall \texttt{x} \in G.\ \imath \bullet \texttt{x} = \texttt{x}
\end{array}$$