First Order versus Higher Order

formulation of axioms

- First Order Axiom of Replacement

```
∀s.
 (∀x∈s. ∀y z. φ(x,y) ∧ φ(x,z) ⇒ y = z)
 ⇒
 ∃t. ∀y. y ∈ t ≡ ∃x∈s. φ(x,y)
```

  - an axiom schema
  - $\phi(x, y)$ ranges over formulae

- Higher Order Axiom of Replacement

```
∀f s. ∃t. ∀y. y ∈ t  =  ∃x∈s. y = f x
```

  - a single term
    - type V
    - constant $\in : V \times V \rightarrow bool$
  - same concept as first order schema