

Programming Logics and Software Verification

Automating Verification:
Procedures,
Hierarchical Data Structures,
Arithmetic Strengthening

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Procedures

Interprocedural analysis

- For each procedure, create a table that records all the past analysis results.
- Table for `create_list()`:

Precond.	Postcondition
emp	$ret=0 \wedge emp,$ $ret \mapsto 0,$ $ls(ret,0)$
...	...

- Use the table whenever possible.

Creation of two lists

```
let create_list() = {...}
```

in

emp

```
x=create_list();
```

```
y=create_list();
```

Creation of two lists

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let create_list() = {...}
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in



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Creation of two lists

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ret=0 \wedge emp,

ret \mapsto 0,

ls(ret,0)

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y=create_list();
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Creation of two lists

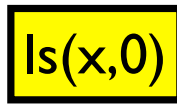
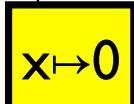
let create_list() = {...}



in



x=create_list();



y=create_list();

Creation of two lists

let create_list() = {...}

emp	ret=0 \wedge emp, ret \mapsto 0, ls(ret,0)
x \mapsto 0	
ls x 0	

in

emp

x=create_list();

x=0 \wedge emp

x \mapsto 0

ls(x,0)

y=create_list();

Creation of two lists

let create_list() = {...}

emp	ret=0 \wedge emp,	ret \mapsto 0,	ls(ret,0)
x \mapsto 0	ret=0 \wedge x \mapsto 0,	x \mapsto 0 * ret \mapsto 0,	x \mapsto 0 * ls(ret,0)
ls x 0	ret=0 \wedge ls(x,0),	ls(x,0) * ret \mapsto 0,	ls(x,0) * ls(ret,0)

in

emp

x=create_list();

x=0 \wedge emp

x \mapsto 0

ls(x,0)

y=create_list();

Creation of two lists

let create_list() = {...}

emp	ret=0 \wedge emp,	ret \mapsto 0,	ls(ret,0)
x \mapsto 0	ret=0 \wedge x \mapsto 0,	x \mapsto 0 * ret \mapsto 0,	x \mapsto 0 * ls(ret,0)
ls x 0	ret=0 \wedge ls(x,0),	ls(x,0) * ret \mapsto 0,	ls(x,0) * ls(ret,0)

in

emp

x=create_list();

x=0 \wedge emp

x \mapsto 0

ls(x,0)

y=create_list();

x=0 \wedge y=0 \wedge emp

x=0 \wedge y \mapsto 0

.....

x \mapsto 0 * y \mapsto 0

.....

ls(x,0) * ls(y,0)

Creation of two lists

let create_list() = {...}

emp	ret=0 \wedge emp,	ret \mapsto 0,	ls(ret,0)
x \mapsto 0	ret=0 \wedge x \mapsto 0,	x \mapsto 0 * ret \mapsto 0,	x \mapsto 0 * ls(ret,0)
ls x 0	ret=0 \wedge ls(x,0),	ls(x,0) * ret \mapsto 0,	ls(x,0) * ls(ret,0)

3 entries, 9 results

in

emp

x=create_list();

x=0 \wedge emp

x \mapsto 0

ls(x,0)

y=create_list();

x=0 \wedge y=0 \wedge emp

x=0 \wedge y \mapsto 0

.....

x \mapsto 0 * y \mapsto 0

.....

ls(x,0) * ls(y,0)

Creation of two lists

let create_list() = {...}

emp	ret=0 \wedge emp,	ret \mapsto 0,	ls(ret,0)
x \mapsto 0	ret=0 \wedge x \mapsto 0,	x \mapsto 0 * ret \mapsto 0,	x \mapsto 0 * ls(ret,0)
ls x 0	ret=0 \wedge ls(x,0),	ls(x,0) * ret \mapsto 0,	ls(x,0) * ls(ret,0)

in

3 entries, 9 results

The verifier constructs proofs of two Hoare triples for create_list unnecessarily.

y=create_list();

x=0 \wedge y=0 \wedge emp

x=0 \wedge y \mapsto 0

.....

x \mapsto 0 * y \mapsto 0

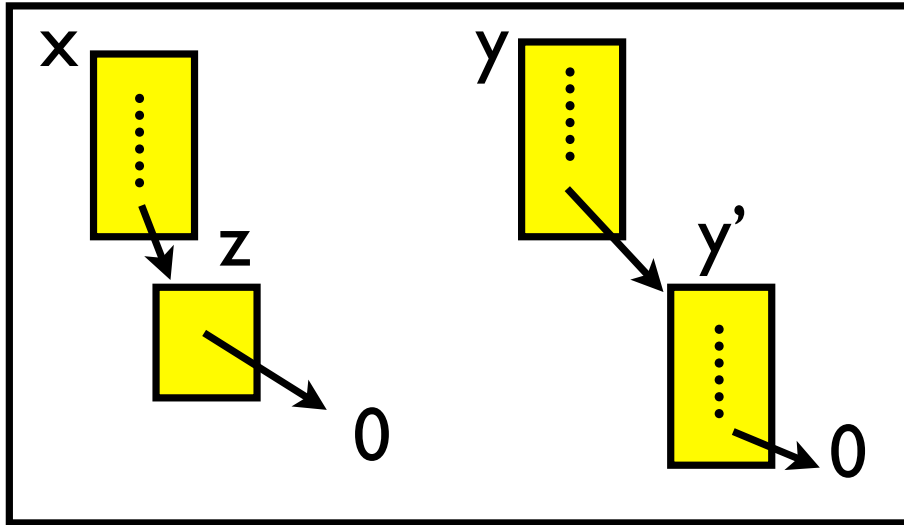
.....

ls(x,0) * ls(y,0)

Optimisation by the frame rule

- Pass & change only the part of a symbolic heap, that is reachable from the parameters. [Rinetzky et al., Gotsman et al.]
- E.g.

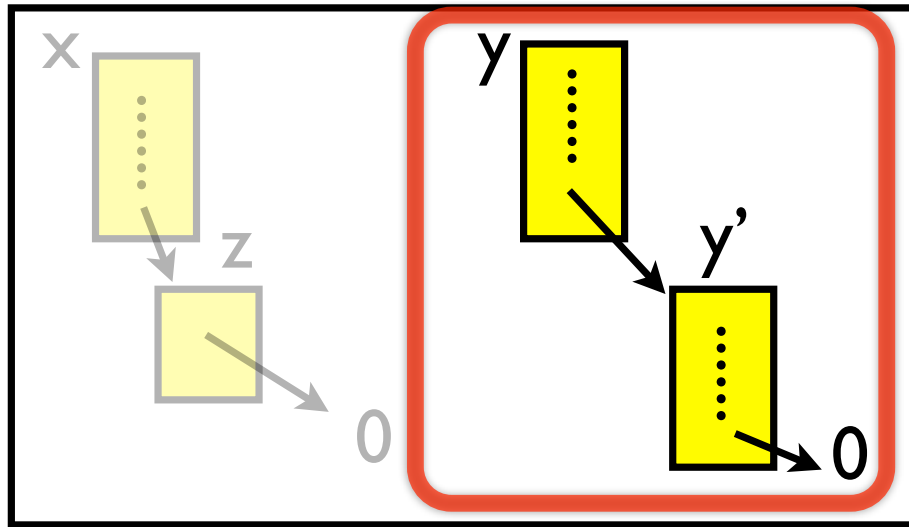
$$\{\exists y'. \text{ls}(x,z) * z \mapsto 0 * \text{ls}(y,y') * \text{ls}(y',0)\} \text{dispose_list}(y)$$



the frame rule

- Pass & change only the part of a symbolic heap, that is reachable from the parameters. [Rinetzky et al., Gotsman et al.]
- E.g.

$\{\exists y'. ls(x,z) * z \mapsto 0 * ls(y,y') * ls(y',0)\} \text{ dispose_list}(y)$



the frame rule

- Pass & change only the part of a symbolic heap, that is reachable from the parameters. [Rinetzky et al., Gotsman et al.]
- E.g.

$$\{\exists y'. ls(x,z) * z \mapsto 0 * ls(y,y') * ls(y',0)\} \text{ dispose_list}(y)$$

Creation of 2 Lists

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let create_list() = {...}
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Creation of 2 Lists

```
let create_list() = {...}
```



```
in
```



```
x=create_list();
```

```
y=create_list();
```

Creation of 2 Lists

```
let create_list() = {...}
```

emp

ret=0 \wedge emp,

ret \mapsto 0,

ls(ret,0)

in

emp

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x=create_list();
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y=create_list();
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Creation of 2 Lists

let create_list() = {...}



in

emp

x=create_list();

x=0 \wedge emp

x \mapsto 0

ls(x,0)

y=create_list();

Creation of 2 Lists

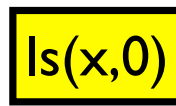
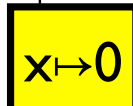
let create_list() = {...}



in



x=create_list();



y=create_list();

Creation of 2 Lists

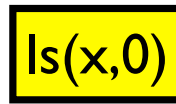
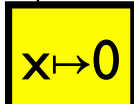
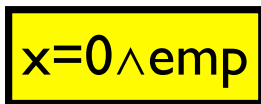
let create_list() = {...}



in



x=create_list();



y=create_list();

Creation of 2 Lists

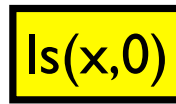
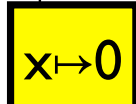
let create_list() = {...}



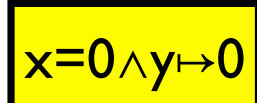
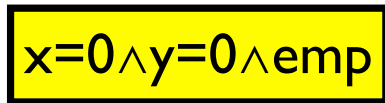
in



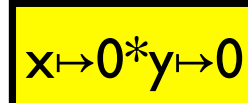
x=create_list();



y=create_list();



.....



.....



Creation of 2 Lists

let create_list() = {...}



~~3 entries, 9 results~~

1 entries, 3 results

in

emp

x=create_list();

x=0 \wedge emp

x \mapsto 0

ls(x,0)

y=create_list();

x=0 \wedge y=0 \wedge emp

x=0 \wedge y \mapsto 0

.....

x \mapsto 0 * y \mapsto 0

.....

ls(x,0) * ls(y,0)

Challenges for interprocedural analyses – efficiency

- ▶ Underlying intraprocedural analysis should be efficient
- ▶ Interprocedural analyses compute summaries and reuse them:

$\{ls(x, nil) * ls(y, nil)\}$ $\{ls(u, nil) * ls(v, nil)\}$

append(x, y) append(u, v)

$\{ls(x, y) * ls(y, nil)\}$ $\{ls(u, v) * ls(v, nil)\}$

- ▶ Efficiency \Rightarrow more reusable summaries needed:

$\{ls(x, nil) * ls(y, nil)\}$ $\{ls(x, nil) * ls(y, nil) * ls(z, nil)\}$

append(x, y) append(x, y)

$\{ls(x, y) * ls(y, nil)\}$ $\{?\}$

- ▶ Procedures should be analyzed on local heaps: $ls(x, nil)$

Challenges for interprocedural analyses – precision

Summary: $\{ls(x, nil) * ls(y, nil)\}$ `append(x, y)` $\{ls(x, y) * ls(y, nil)\}$

$\{ls(x, nil) * ls(y, nil) * ls(z, nil)\}$

`append(x, y)`

$\{ls(x, y) * ls(y, nil) * ls(z, nil)\}$

`append(x, z)`

$\{?\}$

- ▶ y – (heap) cutpoint (Rinetzky et al., POPL'05)

Challenges for interprocedural analyses – precision

Summary: $\{ls(x, nil) * ls(y, nil)\}$ `append(x, y)` $\{ls(x, y) * ls(y, nil)\}$

$\{ls(x, nil) * ls(y, nil) * ls(z, nil)\}$

`append(x, y)`

$\{ls(x, y) * ls(y, nil) * ls(z, nil)\}$

`append(x, z)`

$\{ls(x, y) * ls(y, z) * ls(z, nil)\}$

- ▶ y – (heap) cutpoint (Rinetzky et al., POPL'05)
- ▶ Handling cutpoints precisely and efficiently is difficult

Handling procedure calls and returns

- ▶ Need to compute local heap,
- ▶ ...analyze procedure on the local heap,
- ▶ ...and recombine the result with the rest of the heap.

Handling procedure calls and returns

- ▶ Need to compute local heap,
- ▶ ...analyze procedure on the local heap,
- ▶ ...and recombine the result with the rest of the heap.
- ▶ Idea: use the FRAME rule

$$\frac{\text{FRAME} \quad \{P\} C \{Q\}}{\{P * R\} C \{Q * R\}} \quad C \text{ does not modify variables in } R$$

- ▶ Example:

$\{ls(x, nil) * ls(y, nil)\} \text{ append}(x, y) \{ls(x, y) * ls(y, nil)\}$

\Rightarrow

$\{ls(x, nil) * ls(y, nil) * ls(z, nil)\}$

$\text{append}(x, y)$

$\{ls(x, y) * ls(y, nil) * ls(z, nil)\}$

Local procedure call rule

$$\text{LOCALPROC CALL} \quad \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

Local procedure call rule

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

1. Given a heap S at the call-site of $f(\vec{x}\sigma)$

Local procedure call rule

$$\text{LOCALPROC CALL} \frac{S \vDash P\sigma * R \quad Q\sigma * R \vDash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

1. Given a heap S at the call-site of $f(\vec{x}\sigma)$
2. Split S into: $P\sigma$ (a local heap) and R (a frame)

Local procedure call rule

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1. Given a heap S at the call-site of $f(\vec{x}\sigma)$
2. Split S into: $P\sigma$ (a local heap) and R (a frame)
3. Express the pre-heap $P\sigma$ in terms of the formal parameters: P

Local procedure call rule

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4. Compute the post-heap of the procedure call on P : Q

Local procedure call rule

$$\text{LOCALPROC CALL} \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

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3. Express the pre-heap $P\sigma$ in terms of the formal parameters: P
4. Compute the post-heap of the procedure call on P : Q
5. Express Q in terms of the actual parameters: $Q\sigma$

Local procedure call rule

$$\frac{\text{LOCALPROC CALL} \quad S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

1. Given a heap S at the call-site of $f(\vec{x}\sigma)$
2. Split S into: $P\sigma$ (a local heap) and R (a frame)
3. Express the pre-heap $P\sigma$ in terms of the formal parameters: P
4. Compute the post-heap of the procedure call on P : Q
5. Express Q in terms of the actual parameters: $Q\sigma$
6. *-conjoin $Q\sigma$ with the frame R , yielding the post-heap: T

Local procedure call rule

$$\text{LOCALPROC CALL}$$
$$\frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, nil) * ls(v, nil) * ls(w, nil)\} \text{append}(u, v) \{?\}$

Local procedure call rule

$$\text{LOCALPROC CALL} \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, nil)$ and $ls(v, nil) * ls(w, nil)$

Local procedure call rule

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, nil)$ and $ls(v, nil) * ls(w, nil)$

$ls(x, nil)$

Local procedure call rule

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, nil)$ and $ls(v, nil) * ls(w, nil)$

$ls(x, nil)$

$ls(x, y)$

Local procedure call rule

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, nil)$ and $ls(v, nil) * ls(w, nil)$

$ls(x, nil)$

$ls(x, y)$

$ls(u, v)$

Local procedure call rule

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, nil)$ and $ls(v, nil) * ls(w, nil)$

$ls(x, nil)$

$ls(x, y)$

$ls(u, v)$

$ls(u, v) * ls(v, nil) * ls(w, nil)$

Choice of heap splittings

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

- ▶ $\{ls(u, nil) * ls(v, nil) * ls(w, nil)\}$ `append(u, v)` {?}
- ▶ Split S into a local heap and a frame:
 - ▶ `emp` and $ls(u, nil) * ls(v, nil) * ls(w, nil)$
 - ▶ $ls(u, nil)$ and $ls(v, nil) * ls(w, nil)$
 - ▶ $ls(u, nil) * ls(v, nil)$ and $ls(w, nil)$
 - ▶ $ls(u, nil) * ls(v, nil) * ls(w, nil)$ and `emp`
- ▶ All splittings are sound
- ▶ Local heap = the portion of the heap reachable from actual parameters:
 $ls(u, nil) * ls(v, nil)$ and $ls(w, nil)$

Cutpoints

$$\text{LOCALPROC CALL}$$
$$\frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{append}(u, v) \{?\}$

Cutpoints

$$\text{LOCALPROC CALL} \\ \frac{S \vDash P\sigma * R \quad Q\sigma * R \vDash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

Cutpoints

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

$ls(x, ?) * ls(?, nil) * ls(y, nil)$

Cutpoints

$$\text{LOCALPROC CALL} \\ \frac{S \vDash P\sigma * R \quad Q\sigma * R \vDash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

$ls(x, X) * ls(X, nil) * ls(y, nil)$

Cutpoints

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

$ls(x, X) * ls(X, nil) * ls(y, nil)$

$ls(x, X) * ls(X, y) * ls(y, nil)$

Cutpoints

$$\text{LOCALPROC CALL} \\ \frac{S \vDash P\sigma * R \quad Q\sigma * R \vDash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

$ls(x, X) * ls(X, nil) * ls(y, nil)$

$ls(x, X) * ls(X, y) * ls(y, nil)$

$ls(u, c) * ls(c, v) * ls(v, nil)$

Cutpoints

$$\text{LOCALPROC CALL} \\ \frac{S \models P\sigma * R \quad Q\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

$ls(x, X) * ls(X, nil) * ls(y, nil)$

$ls(x, X) * ls(X, y) * ls(y, nil)$

$ls(u, c) * ls(c, v) * ls(v, nil)$

$ls(u, c) * ls(c, v) * ls(v, nil) * ls(w, nil)$

Handling cutpoints

- ▶ Problem: number of unquantified variables is no longer bounded
 - ▶ abstract domain will be infinite!
- ▶ Solution:
 - ▶ bound the number of cutpoints
 - ▶ quantify the excess

Handling cutpoints

$$\frac{\text{LOCALPROC CALLCUT} \quad S \models P' * R \quad \exists \vec{c}. P' \models P_\sigma \quad Q_\sigma * R \models T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

1. Given a heap S at the call-site of $f(\vec{x}\sigma)$
2. Split S into: P_σ (a local heap) and R (a frame)
3. Quantify extra cutpoints and abstract
4. Express the pre-heap P_σ in terms of the formal parameters: P
5. Compute the post-heap of the procedure call on P : Q
6. Express Q in terms of the actual parameters: Q_σ
7. *-conjoin Q_σ with the frame R , yielding the post-heap: T

Handling cutpoints

$$\text{LOCALPROC CALL CUT}$$
$$\frac{S \vDash P' * R \quad \exists \vec{c}. P' \vDash P_\sigma \quad Q_\sigma * R \vDash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

Handling cutpoints

$$\text{LOCALPROC CALL CUT}$$
$$\frac{S \Vdash P' * R \quad \exists \vec{c}. P' \Vdash P_\sigma \quad Q_\sigma * R \Vdash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

Handling cutpoints

$$\text{LOCALPROC CALL CUT}$$
$$\frac{S \Vdash P' * R \quad \exists \vec{c}. P' \Vdash P_\sigma \quad Q_\sigma * R \Vdash T}{\Gamma, \{P\} f(\vec{x}) \{Q\} \vdash \{S\} f(\vec{x}\sigma) \{T\}}$$

$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\} \text{ append}(u, v) \{?\}$

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

$ls(u, c') * ls(c', nil) * ls(v, nil)$

Handling cutpoints

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$\{ls(u, c) * ls(c, nil) * ls(v, nil) * ls(w, nil)\}$ append(u, v) {?}

$ls(u, c) * ls(c, nil) * ls(v, nil)$ and $ls(w, nil)$

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Local variables

- ▶ Problem: callee may have local variables
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$$ls(x, y) * ls(y, nil)$$

Overall analysis — interprocedural analysis

- ▶ Bound on the number of cutpoints handled at a time
- ▶ Procedure calls and returns through `LOCALPROCSCALLCUT`
- ▶ Reps-Horwitz-Sagiv algorithm for tabulating summaries:
 - ▶ start from the first statement of `main()`
 - ▶ go forward and top-down
 - ▶ if the function isn't analyzed for a heap
 - obtain a local heap and analyze it for this heap
 - ▶ else
 - reuse the existing summary
- ▶ Elements of abstract domain: tables of path edges
- ▶ Abstract domain is finite \Rightarrow RHS terminates

Soundness

- ▶ Via compilation to separation logic
- ▶ Run of the analysis \Rightarrow collection of proofs
- ▶ Summary \Rightarrow valid Hoare triple in separation logic
- ▶ Proof by induction on the structure of the program
 - ▶ uses the rule `LOCALPROCALLCUT`

Interprocedural Analysis Summary

- Optimizing using Frame rule beneficial because
 - Most procedures modify only a small subset of the heap
 - Their effect is local
 - Separated heap abstractions mirror this locality
- Structural rules in separation logic often enable sound optimizations
 - Frame rule for interprocedural analysis
 - Concurrency rules and thread-local analysis
- Separation logic proof rules resolve tricky semantic issues
 - Allow optimized symbolic execution

Hierarchical Data Structures

```

void
t1394Diag_CancelIrp(
    IN PDEVICE_OBJECT DeviceObject,
    IN PIRP Irp
)
{
    KIRQL Irql;
    PBUS_RESET_IRP BusResetIrp;
    PDEVICE_EXTENSION deviceExtension;

    ENTER("t1394Diag_CancelIrp");

    deviceExtension = DeviceObject->DeviceExtension;

    KeAcquireSpinLock(&deviceExtension->ResetSpinLock, &Irql);

    BusResetIrp = (PBUS_RESET_IRP) deviceExtension->BusResetIrpList.Flink;

    TRACE(TL_TRACE, ("Irp = 0x%x\n", Irp));

    while (BusResetIrp) {

        TRACE(TL_TRACE, ("Cancelling BusResetIrp->Irp = 0x%x\n", BusResetIrp->Irp));

        if (BusResetIrp->Irp == Irp) {

            RemoveEntryList(&BusResetIrp->BusResetIrpList);
            ExFreePool(BusResetIrp);
            break;

        }
        else if (BusResetIrp->BusResetIrpList.Flink == &deviceExtension->BusResetIrpList) {
            break;
        }
        else
            BusResetIrp = (PBUS_RESET_IRP) BusResetIrp->BusResetIrpList.Flink;

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```

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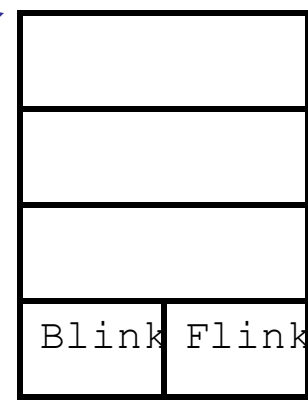
        TRACE(TL_TRACE, ("Cancelling BusResetIrp->Irp = 0x%x\n", BusResetIrp->Irp));

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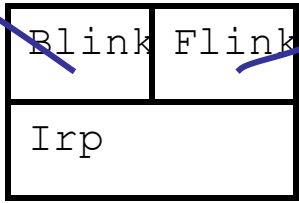
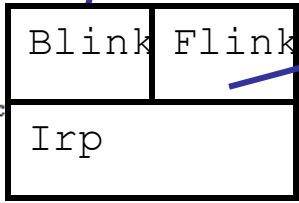
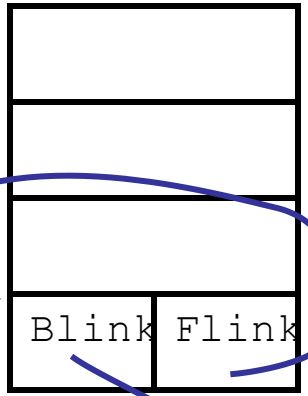
            RemoveEntryList(&BusResetIrp->BusResetIrpList);
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            break;

        }
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    }
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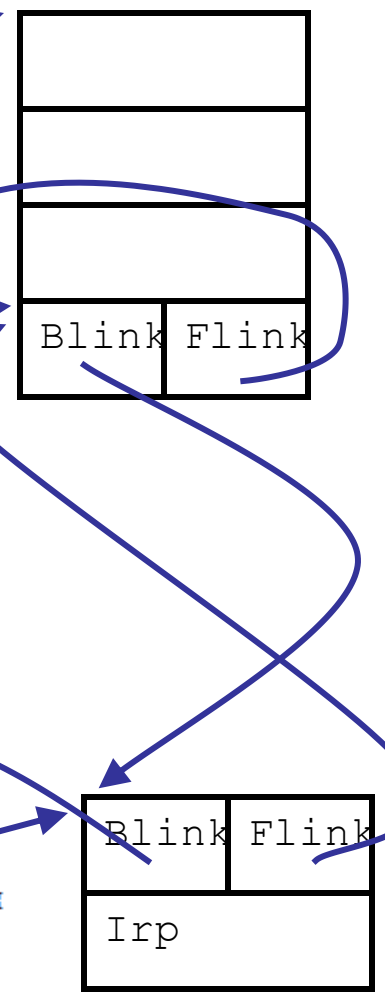
```



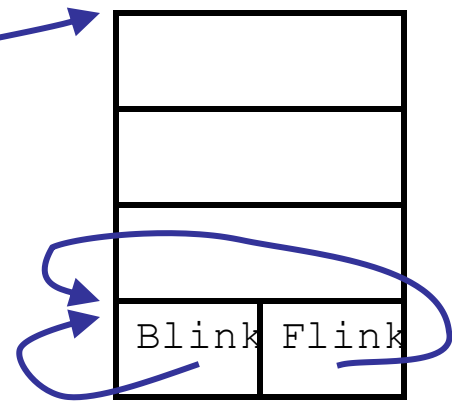
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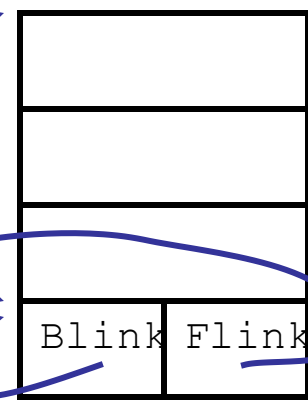
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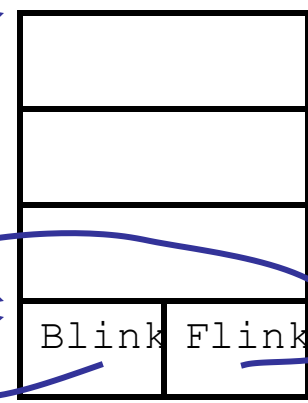
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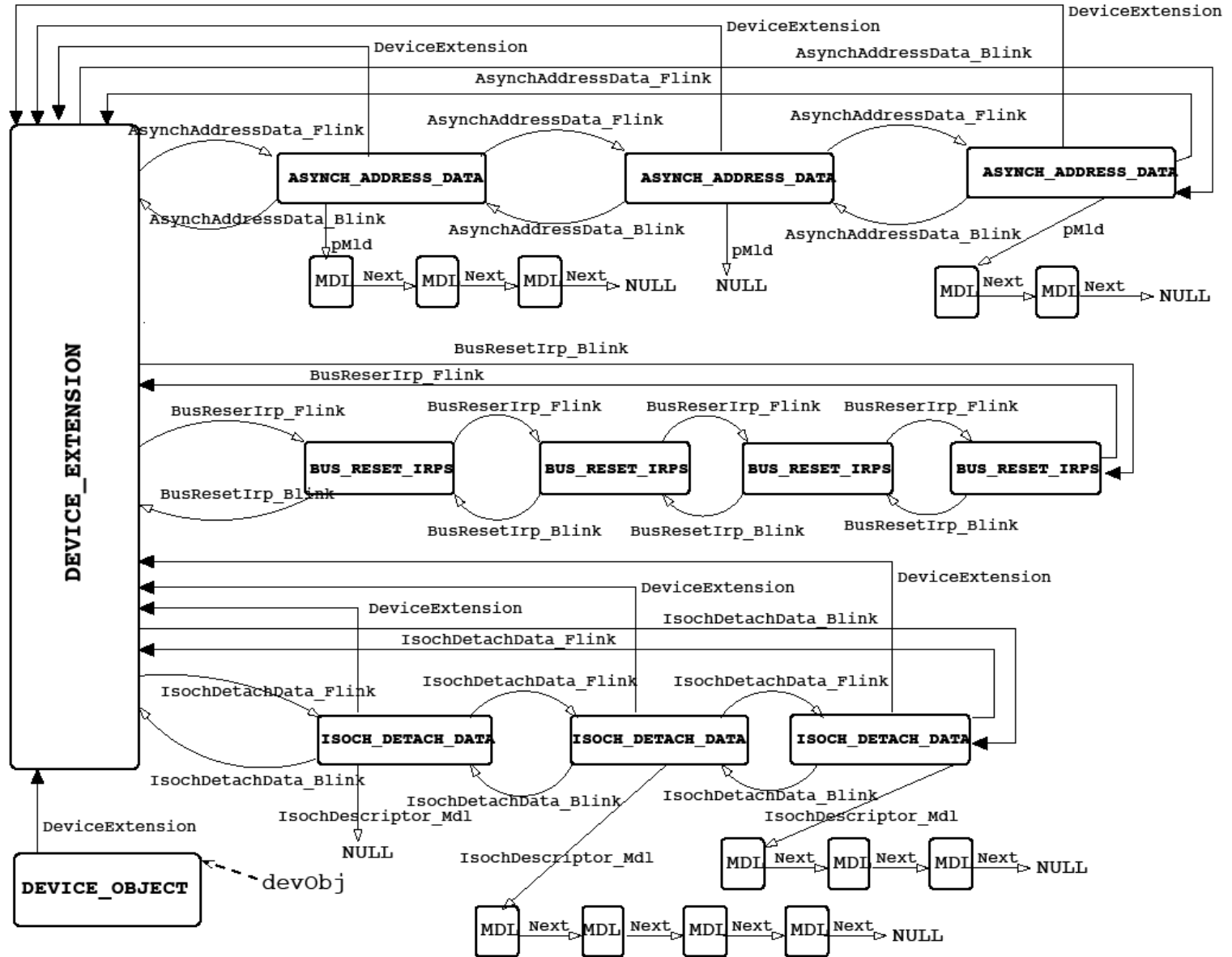
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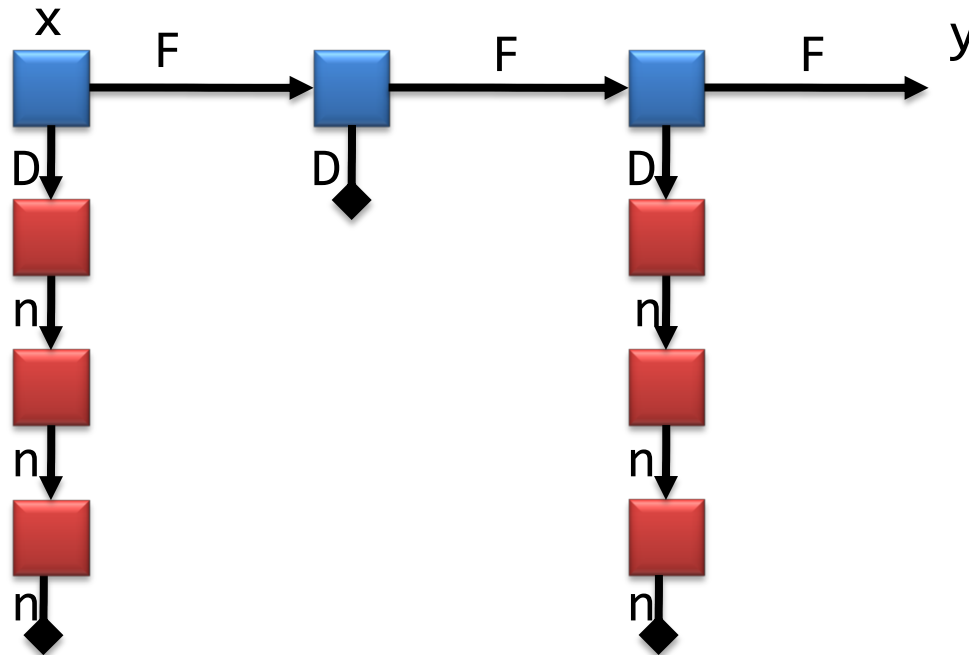


Second-Order Linked List Segments

- $\text{hls}(\Delta, E, F) \Leftrightarrow (E=F \wedge \text{emp})$
 $\vee (\exists y'. \Delta(E, y') * \text{hls}(\Delta, y', F))$

- For: $\Delta\text{ls}(x, y) = \exists z'. (x \mapsto \{F:y, D:z'\}) * \text{ls}(z', \text{null})$

$\text{hls}(\Delta\text{ls}, x, y)$

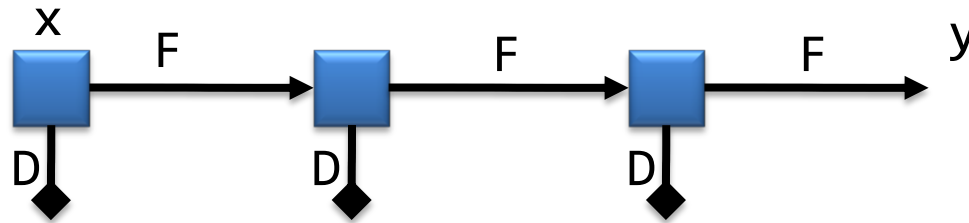


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 $\vee (\exists y'. \Delta(E, y') * \text{hls}(\Delta, y', F))$

- For: $\Delta \text{emp}(x, y) = \exists z'. (x \mapsto \{F:y, D:z'\})$

$\text{hls}(\Delta \text{emp}, x, y)$



Symbolic Execution with HO lists

```
trim(x,y) {  
  {hls( $\Delta$ ls, x, y)}  
  if (x!=y) {  
    { $\exists w',z'. (x \mapsto \{F:w', D:z'\}) * \text{ls}(z', \text{null}) * \text{hls}(\Delta\text{ls}, w', y)$ }  
    free_list(x->D);  
    { $\exists w',z'. (x \mapsto \{F:w', D:z'\}) * \text{emp} * \text{hls}(\Delta\text{ls}, w', y)$ }  
    { $\exists w',z'. (x \mapsto \{F:w', D:z'\}) * \text{hls}(\Delta\text{ls}, w', y)$ }  
    trim(x->F,y);  
    { $\exists w',z'. (x \mapsto \{F:w', D:z'\}) * \text{hls}(\Delta\text{emp}, w', y)$ }  
  }  
  {hls( $\Delta\text{emp}$ , x, y)}  
}
```

Supposing: $\{\text{ls}(x, \text{null})\}$ free_list(x); {emp}

Hierarchical Data Structures

- Adaptive shape analysis
 - build in induction principles, rather than particular data structures
 - automatic recognition of many complex variations on linked lists:
 - singly-linked list segments
 - ...of non-empty doubly-linked lists
 - ...with back-pointers to the head node
 - ...of cyclic doubly-linked lists
 - ...