

Deny-Guarantee

(ongoing research)

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RG + SL

RGSep: use rely-guarantee to extend the power of separation logic.

Deny-guarantee: use separation logic to extend the power of rely-guarantee.

Programming language

$C ::= \dots \mid x := \text{fork}(C) \mid \text{join}(E) \mid \dots$

Simple program

```
t1 := fork(x:=false);  
t2 := fork(x:=true);  
join(t1);  
x := true;  
if (x == false) error;
```

Rely-guarantee?

???

$R, G \vdash \{ P \} x := \text{fork}(C) \{ Q \}$

Beyond rely-guarantee

Move the interference into the state:

$$R, G \vdash \{P\} C \{Q\}$$

Beyond rely-guarantee

Move the interference into the state:

$$\{ P \} C \{ Q \}$$

P and Q define:

- what the state holds; and
- what interference can occur

Fork Rule

$$\frac{\{ P \} C \{ Q \}}{\{ P \} t := \text{fork}(C) \{ \text{thr}(t, Q) \}}.$$

Join Rule

$\{ \text{thr}(t, Q) \} \quad \text{join}(t) \quad \{ Q \}$

Assignment

P stable

Q stable

allowed(P, Q)

$P \Rightarrow Q [x=E]$

$\{ P \} x := E \{ Q \}$

Stability

In rely-guarantee, stability is defined with respect to some rely.

In deny-guarantee, we define assertion stability on its own:

$$\text{stable}(P) \iff \sigma \vDash P \wedge (\sigma, \sigma') \vDash \text{allowed}(P) \implies \sigma' \vDash P$$

**How do we keep track of behaviour
between threads?**

Recall: RG Parallel Rule

$$G_1 \Rightarrow R_2$$

$$G_2 \Rightarrow R_1$$

$$R_1, G_1 \vdash \{ P_1 \} C_1 \{ Q_1 \}$$

$$R_2, G_2 \vdash \{ P_2 \} C_2 \{ Q_2 \}$$

$$R_1 \wedge R_2, G_1 \vee G_2 \vdash \{ P_1 \wedge P_2 \} C_1 \parallel C_2 \{ Q_1 \wedge Q_2 \}$$

Interference as resource

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Separation logic to the rescue!

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Separation logic to the rescue!

DG introduces resources called *permissions*, tracking what behaviour is allowed to each thread.

Permissions

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$$[x=0 \rightsquigarrow x=1]_{1/2}$$

$$[x=n \rightsquigarrow x=n+1]_1$$

$$[x=1 \rightsquigarrow x=2]_1 * [x=3 \rightsquigarrow x=4]_{1/4}$$

Permissions

$[x = _ \rightsquigarrow x = \text{true}]_1$: exclusive permission.

$[x = _ \rightsquigarrow x = \text{true}]_f$ s.t. $f < 1$: non-exclusive permission.

Permission splitting

Permissions can be split:

$$[x = _ \rightsquigarrow x = \text{true}]_{1/2} * [x = _ \rightsquigarrow x = \text{true}]_{1/2} \Leftrightarrow [x = _ \rightsquigarrow x = \text{true}]_1$$

Splitting an exclusive permission gives a non-exclusive permission.

Stability in deny-guarantee

Permissions ensure stability:

For example:

$$[x=0 \rightsquigarrow x=1]_{1/2} \wedge x = 1$$

is stable.

Encoding rely-guarantee

Previously:

$$R = (x=0 \rightsquigarrow x=1) = G$$

$$R, G \vdash \{x=1\} \text{ skip } \{x=1\}$$

Now write:

$$\{ [x=0 \rightsquigarrow x=1]_{1/2} \wedge x=1 \}$$

skip

$$\{ [x=0 \rightsquigarrow x=1]_{1/2} \wedge x=1 \}$$

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Thread specifications

$\{ [x = _ \rightsquigarrow x = \text{false}]_1 \}$
 $x := \text{false}$

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Proving the program

$\{ [x=_ \rightsquigarrow x=true]_I * [x=_ \rightsquigarrow x=false]_I \}$

t1 := fork(x:=false);

t2 := fork(x:=true);

join(t1);

x := true;

if (x == false) **error**;

Proving the program

$\{ [x=_ \rightsquigarrow x=true]_I * [x=_ \rightsquigarrow x=false]_I \}$

t1 := fork(x:=false);

$\{ [x=_ \rightsquigarrow x=true]_I * thr(t1, [x=_ \rightsquigarrow x=false]_I) \}$

t2 := fork(x:=true);

join(t1);

x := true;

if (x == false) **error**;

Proving the program

$\{ [x = _ \rightsquigarrow x = \text{true}]_1 * [x = _ \rightsquigarrow x = \text{false}]_1 \}$

t1 := fork(x:=false);

$\{ [x = _ \rightsquigarrow x = \text{true}]_1 * \text{thr}(t1, [x = _ \rightsquigarrow x = \text{false}]_1) \}$

t2 := fork(x:=true);

$\{ [x = _ \rightsquigarrow x = \text{true}]_{1/2} * \text{thr}(t2, [x = _ \rightsquigarrow x = \text{true}]_{1/2}) * \text{thr}(t1, [x = _ \rightsquigarrow x = \text{false}]_1) \}$

join(t1);

x := true;

if (x == false) **error**;

Proving the program

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join(t1);

$\{ [x = _ \rightsquigarrow x = \text{true}]_{1/2} * \text{thr}(t2, [x = _ \rightsquigarrow x = \text{true}]_{1/2}) * [x = _ \rightsquigarrow x = \text{false}]_1 \}$

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Proving the program

$\{ [x = _ \rightsquigarrow x = \text{true}]_1 * [x = _ \rightsquigarrow x = \text{false}]_1 \}$

$t1 := \text{fork}(x := \text{false});$

$\{ [x = _ \rightsquigarrow x = \text{true}]_1 * \text{thr}(t1, [x = _ \rightsquigarrow x = \text{false}]_1) \}$

$t2 := \text{fork}(x := \text{true});$

$\{ [x = _ \rightsquigarrow x = \text{true}]_{1/2} * \text{thr}(t2, [x = _ \rightsquigarrow x = \text{true}]_{1/2}) * \text{thr}(t1, [x = _ \rightsquigarrow x = \text{false}]_1) \}$

$\text{join}(t1);$

$\{ [x = _ \rightsquigarrow x = \text{true}]_{1/2} * \text{thr}(t2, [x = _ \rightsquigarrow x = \text{true}]_{1/2}) * [x = _ \rightsquigarrow x = \text{false}]_1 \}$

$x := \text{true};$

$\{ [x = _ \rightsquigarrow x = \text{true}]_{1/2} * \text{thr}(t2, [x = \text{true}]_{1/2}) * [x = _ \rightsquigarrow x = \text{false}]_1 \wedge x = \text{true} \}$

$\text{if } (x == \text{false}) \text{ error};$

References

- Dodds *et al.* Deny-guarantee Reasoning, ESOP'09.
- Dinsdale-Young *et al.* Concurrent Abstract Predicates, ECOOP'10.