Sorting Workset[1]

Murray Edwards: Due on 7th May, 12:30.
Queens, Robinson: Due on 9th May, at the same time as that of your Thursday supervision

All theoretical exercises - mandatory. Besides those, pick at least one of the implementation challenges.

1. What is the asymptotic complexity of the function that computes the n-th Fibonacci number? Obtain a tight bound. Hint: $O(2^n)$ is not tight, but assume it's still exponential: $\Theta(a^n)$.

2. Compute the asymptotic complexity for a function recursively defined by:

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 5 \cdot T(n/10) + n^2 & \text{if } n > 1 \end{cases}$$

3. **Challenge**: Implement Mergesort using only n/2 additional space - I recommend you make use of the C++ helper project on my site: `http://www.cl.cam.ac.uk/~lc525/supervisions.html`, but you can also choose an implementation language of your choice.

4. 2010, Paper1, Q5: A programmer is tasked with sorting both arrays and linked lists. For both data structures, he intends to use the mergesort algorithm.

   c) ii) The programmer only knows how to merge two arrays in O(n) space and linked lists in O(1) space, so proposes converting the arrays to linked lists before applying the mergesort algorithm to save on space. Comment on this strategy.

5. Problem **8-4** from CLRS (Stack depth for quicksort)

6. **Challenge**: Implement Heapsort (same note as for the first challenge, C++ recommended, any other language accepted)

7. 2011, Paper1, Q5: Generalise the binary min-heap to one where nodes have not 2 but k children.

   (i) State the two defining properties of a min-heap, one constraining the shape and one constraining the keys of the data structure, and describe how to represent a k-ary min-heap as an array.

   (ii) Give a clear description of an algorithm (a simple generalisation of the well-known one for binary heaps) that takes an arbitrary n-item array and efficiently rearranges its elements to turn it into an array representing a k-ary heap.

   (iii) Analyse its complexity as a function of n and k.

[1] Late submissions will only be accepted on special circumstances, and have to be announced at least one day before the deadline, by email at lc525@cam.ac.uk