1. Prove the following two statements, using any method you like.
   (a) \( p \rightarrow q \not\equiv q \rightarrow p \)
   (b) \( (p \rightarrow q) \rightarrow ((\neg p \rightarrow q) \rightarrow q) \simeq \top \)

2. Reduce each of the following formulas to NNF, CNF, and DNF.
   (a) \( (q \rightarrow \neg p) \rightarrow p \)
   (b) \( ((p \land q) \lor r) \land \neg (p \lor r) \)

3. For each formula \( \varphi \) below, either prove that it is valid, or give an interpretation satisfying \( \neg \varphi \) :
   (a) \( (p \rightarrow r) \land (q \rightarrow r) \rightarrow (p \lor q \rightarrow r) \)
   (b) \( ((p \rightarrow q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r)) \)
   (c) \( \neg p \lor (q \rightarrow p) \rightarrow (\neg p \land q) \)

4. Prove the following sequents, using the sequent calculus rules given in the notes:
   (a) \( \neg\neg p \Rightarrow p \)
   (b) \( (p \land q) \land r \Rightarrow p \land (q \land r) \)
   (c) \( (p \lor q) \land (p \lor r) \Rightarrow p \lor (q \land r) \)
   (d) \( \neg (p \lor q) \Rightarrow \neg p \land \neg q \)
5. Suppose \( \mathcal{L} \) is a first-order language with symbols \( F \) and \( M \), of arity 2, and \( Ed \), of arity 0. Let \( F(x, y) \) mean that \( x \) is the father of \( y \), and let \( M(x, y) \) mean that \( x \) is the mother of \( y \). The constant \( Ed \) denotes the person Ed. Write for each of the sentences below, a formula of \( \mathcal{L} \) with the same meaning:

(a) Everybody has a mother.
(b) Everybody has a mother and a father.
(c) Whoever has a mother has a father.
(d) Ed is a grandfather.
(e) Nobody’s grandmother is anybody’s father.

6. Consider the formula

\[
\varphi = \forall x \exists y \exists z (P(x, y) \land P(z, y) \land (P(x, z) \rightarrow P(z, x))).
\]

Which of the following models satisfy \( \varphi \)? Justify briefly.

(a) \( \mathcal{M} = (\mathbb{N}, I) \) where \( I[P] = \{(m, n) \mid m, n \in \mathbb{N} \land m < n\} \)
(b) \( \mathcal{M}' = (\mathbb{N}, I) \) where \( I[P] = \{(m, 2 \cdot m) \mid m \in \mathbb{N}\} \)
(c) \( \mathcal{M}'' = (\mathbb{N}, I) \) where \( I[P] = \{(m, n) \mid m, n \in \mathbb{N} \land m < n + 1\} \)