4 Discrete Maths Seminars

Exercise 4.1
(i) Let X and Y be sets. Show there is a bijection between the set of functions $(X \to \mathcal{P}(Y))$ and the set of relations $\mathcal{P}(X \times Y)$.

(ii) Show for any sets X and Y, with Y containing at least two elements, that there cannot be a bijection between X and the set of functions $(X \to Y)$. $\square$

Exercise 4.2 The set of well-bracketed strings is the subset of strings over symbols [and] defined inductively as follows:

> [ ] is well-bracketed;
>
> if x is well-bracketed, then $[x]$ is well-bracketed;
>
> if x and y are well-bracketed, then xy is well-bracketed.

State the principle of rule induction for well-bracketed strings. Prove the number of left brackets [equals the number of right brackets] in any well-bracketed string. $\square$

Exercise 4.3 A simple language is defined with symbols a and b. The grammar of this language has the rules:

- ab is a word;
- if ax is a word, then axx is a word (where x is any string of symbols);
- if $abbbx$ is a word, then ax is a word.

(i) Is $abbbbb$ a word? Either exhibit a derivation, or prove there isn't one.

(ii) Is $abbb$ a word? Either exhibit a derivation, or prove there isn't one.

(iii) Characterise the strings which are words. Prove your characterisation is correct.

$\square$

Exercise 4.4 There are five equally-spaced stepping stones in a straight line across a river. The distance d from the banks to the nearest stone is the same as that between the stones. You can hop distance d or jump $2d$. So for example you could go from one river bank to the other in 6 hops. Alternatively you might first jump, then hop, then jump, then hop. How many distinct ways could you cross the river (you always hop or jump forwards)?

Describe how many distinct ways you could cross a river with n similarly spaced stepping stones. $\square$