3 Discrete Maths Seminars

In the following exercises you may use any results of the lecture notes provided you state them clearly.

Exercise 3.1

(i) Show that the set of all *finite* subsets of $\mathbb{N}$ is countable.

(ii) Show that the set $\mathbb{Q}$ of all rational numbers is countable.

(ii) Show that the set of irrational numbers is uncountable.

$\square$

Exercise 3.2 Which of the following sets are finite, which are infinite but countable, and which are uncountable?

(i) $\{f : \mathbb{N} \to \{0, 1\} \mid \forall n \in \mathbb{N}.\ f(n) \leq f(n+1)\}$

(ii) $\{f : \mathbb{N} \to \{0, 1\} \mid \forall n \in \mathbb{N}.\ f(2n) \neq f(2n+1)\}$

(iii) $\{f : \mathbb{N} \to \{0, 1\} \mid \forall n \in \mathbb{N}.\ f(n) \neq f(n+1)\}$

(iv) $\{f : \mathbb{N} \to \mathbb{N} \mid \forall n \in \mathbb{N}.\ f(n) \leq f(n+1)\}$

(v) $\{f : \mathbb{N} \to \mathbb{N} \mid \forall n \in \mathbb{N}.\ f(n) \geq f(n+1)\}$

$\square$

Exercise 3.3
(i) Show that if a nonempty set A is countable, then there is a surjection $f : \mathbb{N} \to A$.
(ii) Show that if there is a surjection $f : \mathbb{N} \to A$, where A is a set, then A is countable. [Hint: Produce an injection from A to $\mathbb{N}$ by considering the least n for which $f(n) = a$.] $\square$

Exercise 3.4

(i) Show that $\mathbb{Q} \times \mathbb{Q}$ is countable.

(ii) Deduce that any set of disjoint discs (*i.e.* circular areas which may or may not include their perimeter) in the plane $\mathbb{R} \times \mathbb{R}$ is countable. [You may assume that the rational numbers are dense in the real numbers in the sense that for any reals $r_1 < r_2$ there is a rational q such that $r_1 < q < r_2$.]

$\square$