

A NEW BABYLONIAN AND ITALIAN HOURS SUNDIAL SELWYN COLLEGE, CAMBRIDGE

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The new Selwyn Dial was inaugurated in March 2010 and a non-technical description¹ published on the Internet generated considerable interest. Several readers commented that incorporating an introduction to Babylonian and Italian hours in a Bulletin article would be appreciated. Others commented that it was interesting to see photographs taken in the workshop. What follows takes these comments into account. There is also some emphasis on the way the design evolved.

Genesis

Once in while, deeply buried in the mire of administrative chores and general spam, an email arrives which turns out to be a golden nugget.

On 5 November 2008 a message appeared in my inbox from Professor Richard Bowring, the Master of Selwyn College, Cambridge. It began:

Dear Dr King

Selwyn has received a benefaction which
has one string attached to it; we are to
have a sundial...

There was an invitation to lunch to discuss possibilities but no mention of a budget or any other constraints. I accepted the invitation and made a covert reconnaissance visit.

Selwyn College first opened to undergraduates in 1882 and its oldest buildings date from that time. In recent decades the College has expanded considerably and there are numerous south-facing walls to regard as candidate sundial sites.

There are also many paved areas and lawns. These are potential sites for analemmatic sundials or sundials on pedestals but, in Cambridge, college lawns are treated as hallowed ground. Only Fellows and gardeners are permitted to walk on them; a sundial would be enjoyed by only a select few.

The original court, Old Court, is still the heart of the College and this seemed to be the best place to explore. The court follows a traditional plan with a roughly square lawn surrounded by paving. Three sides are dominated by the gatehouse, chapel and hall. The fourth side, the north side, is largely student accommodation.

I retired to my office and, courtesy of Google Earth, established that the north side of Old Court declines about 4° west.

I had recently returned from a short trip to Italy where I had had breakfast each day in sight of a dial that indicated Italian hours (hours since the most recent sunset). I wondered how such a dial would look on the north range of Old Court.

I made an outline sketch and was unable to resist the temptation of adding Babylonian hours (hours since the most recent sunrise). See Fig. 1. I included this sketch in a portfolio of photographs, diagrams and props that I proposed to take with me to lunch.

The First Sketch

Over lunch I described a number of sundial types and expressed my own preference for vertical dials over horizontal dials on two pragmatic grounds. First, horizontal dials are harder to read at a distance and, secondly, they suffer from rain and general detritus, not to mention a problem alluded to in one of Hillaire Belloc's less well-known sundial mottoes:

*I am a Sundial. Ordinary words
Cannot express my thoughts on Birds.*

I explained my liking for wall dials that dominate public squares in Italy and, more locally, for the wall dial in Queens' College just down the road. After lunch, I suggested a spot on the north range of Old Court that would be ideal. Nevertheless, there was early enthusiasm for an analemmatic sundial which could be incorporated into the Old Court paving.

The most suitable site had many merits but it was near a passage that led to another court. In use, a dial in this position could interfere with the free flow of people walking about. I handed over an outline design for an analemmatic sundial but I also handed over my first sketch.

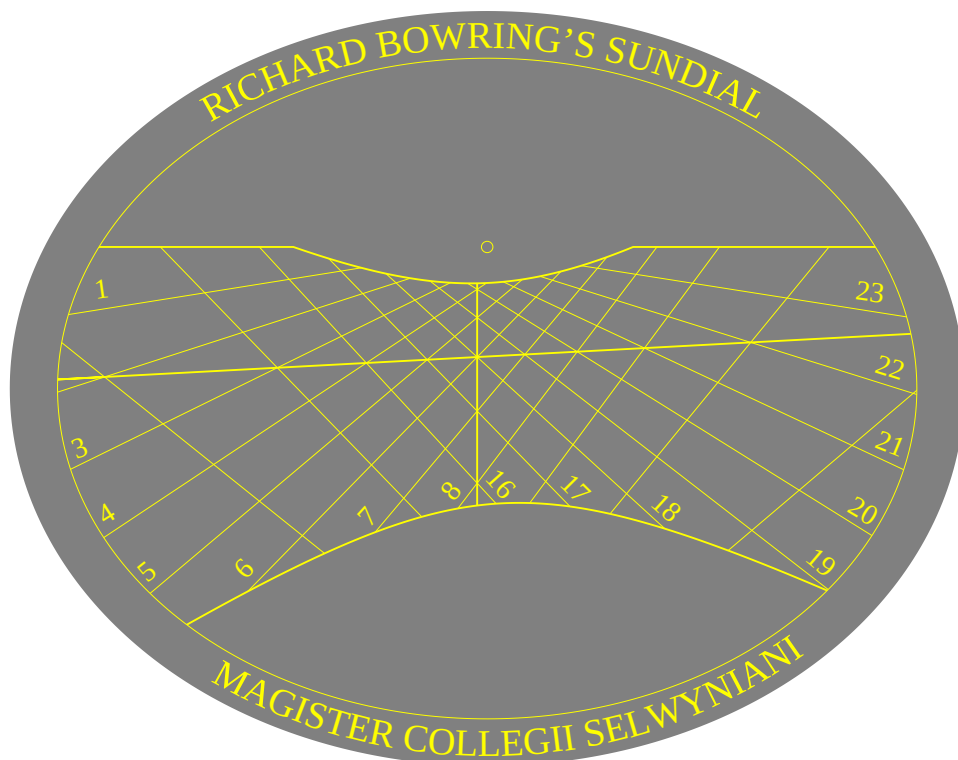


Fig. 1 — A First Sketch

Most of the dials that I have had a hand in have been executed on slate and the principal purpose of this sketch was to give just a hint of the elegance of gilded slate. Yellow lines and poorly aligned lettering on a dark-grey background are a pale imitation of the real thing but they give the right idea.

I chose an elliptical shape because it suited my favoured site, a blank area of brickwork between the corners of four windows.

A secondary purpose of the sketch was to show that sundials can readily incorporate inscriptions. Selwyn has a strong tradition in the classical languages so a mixture of English and Latin was used for the sample wording. At this stage the name of the benefactor had not been made public so I referred instead to my host, the Master of Selwyn College.

The hour-lines on my sketch were originally intended as sample gnomonic features but, by now, my own enthusiasm for a real dial that indicated Babylonian and Italian hours had grown. Somehow I had to convey this enthusiasm to the College. I stressed that a sundial based on my first sketch could be both elegant and educational, very appropriate for a Cambridge College.

Changing Perceptions

It is unfortunate that so many people, chancing upon a sundial, compare the indicated time with the time on their watches. The result generally reinforces the Belloc view that sundials make a botch of what is done far better by a watch.

I suggested that a prospective sundial owner should consider choosing a design that accurately indicates a time which is so different from ordinary clock time that there is no danger of being confused with it. As Chris Lusby Taylor² almost said at the BSS Exeter Conference:

*I am a Sundial. I'm prepared to bet
Your watch can't tell when the sun will set.*

I also outlined some history, noting that the introduction of mechanical clocks had forced a change from using unequal hours to equal hours. These clocks ran from 0h to 24h but midnight was not necessarily the best moment to start and end the day. It is rather difficult to estimate the moment of midnight unless you are a trained astronomical observer.

Running a 24-hour day from noon to noon was a possibility. Greenwich Mean Time itself used this scheme until 1926. Until then, the dates in navigators' logs changed at noon and not at midnight.

The traditional day had begun at sunrise and ended at sunset and these were two other obvious possible reference moments. One advantage of using sunrise or sunset is that these moments can be estimated, albeit only approximately, even when the sky is overcast.

The new clocks with their new ways of indicating time spurred the development of new designs of sundial, for sundials were the most convenient way of setting mechanical clocks.

The Cardozo Kindersley Workshop

Although a decision on the choice of sundial type took a few days, the choice of workshop to make the dial was made very quickly. Professor Bowring himself proposed the Cardozo Kindersley Workshop³ in Cambridge.

It was in 1980 that David Kindersley first asked my advice about a sundial (a double wall-dial for a private client in Hungerford) and I have have served as the Workshop Diallist

for numerous dials since. Following David's death in 1995, the Workshop has been run by his wife and business partner Lida Cardozo Kindersley.

Commemorative plaques made by the Cardozo Kindersley Workshop abound in Cambridge and it turned out that Selwyn was the only College that did not have anything made in the Workshop. This sundial would fill a lacuna.

The Slate and the Nodus

Once it had been decided that there would be a slate sundial showing Babylonian and Italian hours, a number of dimensions had to be specified or determined:

1. The size (and shape) of the slate
2. The size (and shape) of the nodus and the nodus height
3. The latitude and declination of the wall

For much of the year, the range of solar azimuth during the course of a day greatly exceeds 180° . In simple terms, the shadow of any object cast onto a wall races in from infinity in the west in the early morning, then crawls along during the middle of the day, before racing out to infinity in the east in the evening.

For any real dial, there will be periods when the shadow is off the dial to the west in the morning and off the dial to the east in the evening. This suggests that the dial should be as wide as convenient, to be landscape rather than portrait.

A rectangle would be fine but the proximity of the corners of two first-floor windows, either side of the intended site for the dial, limits the width unless the upper corners of the rectangle are rounded off. Further rounding-off suggested an ellipse, one of the few decisions which didn't change from the first sketch.

As delivered by Ivett & Reed, a local supplier of stone, the ellipse had major and minor axes of $1067\text{ mm} \times 864\text{ mm}$. This is about as large a slate as can be readily handled. It was 35 mm thick and weighed around 76 kg. It is standard workshop practice, whenever such a slate is turned over or lifted from a table to an easel, to wear safety boots.

On the celestial sphere, Babylonian hour-lines and Italian hour-lines are great circles and these project into separate families of straight lines on a plane dial. The lines in each family do not meet at a point so a gnomon whose shadow indicates a line on the dial cannot be used. Instead a nodus whose shadow indicates a point on the dial is required.

Here there are two design considerations that do not apply to a sundial which has a conventional polar-oriented gnomon. You have to design a nodus and an appropriate support, and you have to choose the nodus height, the perpendicular distance of the nodus from the dial. The sub-nodus point is shown as a yellow circle in Fig. 1.

The nodus was designed as a simple disc, 27 mm in diameter and 2 mm thick, almost exactly the size of a £2 coin. It was supported by a 9 mm-diameter rod set in a hemispherical base. The nodus height is 175 mm. All this was fabricated in brass by Mackay Engineering of Cambridge and the newly-delivered nodus is shown in Fig. 2 casting a trial shadow onto my slate-coloured document case.

The nodus height governs the scale of the gnomonic features on the dial. If the height is reduced, the diamonds of the criss-cross pattern close up. The design looks more cramped

and it becomes more difficult to read the times but the shadow of the nodus is off the dial to the west or east for a smaller proportion of the day.

The chosen nodus height, 175 mm, is a compromise. In summer, the shadow is on the wall but off the dial for about an hour in the morning and another hour in the afternoon. Around the summer solstice, there are further prolonged early morning and late afternoon periods when the sun is on the north side of the wall.



Fig. 2 — The Nodus and Nodus Support

No point on the dial is more than around 600 mm from the base of the nodus support and this figure and the nodus height of 175 mm were used to determine the diameter of the disc nodus. If the nodus is too small its shadow, at extreme points on the dial, degenerates into pure penumbra and does not show up very well.

One needs to imagine a bug, equipped with heavy-duty sunglasses, at the 600 mm point staring at the nodus. The nodus is 625 mm away because, by chance, (175,600,625) is a Pythagorean triplet. To the bug, the disc appears as an ellipse whose major axis matches the diameter of the disc but whose minor axis is reduced by a factor of 175/625 (the sine of the angle subtended by the nodus support).

If the minor axis subtends an angle less than the angular diameter of the sun, the bug will be able to see the limb of the sun on either side of the minor axis when the sun aligns with the nodus. The bug will not be in full shadow.

The chosen diameter for the disc is 27 mm. When seen by the bug as an ellipse, the minor axis is 7.56 mm. At a distance of 625 mm this subtends an angle of about 0.7° , comfortably greater than the angular diameter of the sun, about half a degree. Note that the shadow of a circular nodus mounted parallel to a plane dial will always be a circle, albeit modified by the surrounding penumbra.

There are numerous ways of determining the latitude and orientation of a wall. Preliminary values had already been determined via Google Earth to produce the first sketch. For more refined values, it was decided to use a local company, Hurst Surveys of Toft, who had previously undertaken topographic surveys for the College. By a delightful coincidence, one line of the postal address of Hurst Surveys is Meridian Court. Their premises are very close to the Greenwich Meridian.



Fig. 3 — Surveying the Wall using GPS

Hurst Surveys use highly specialised surveying kit which exploits the Global Positioning System (GPS). In the photograph in Fig. 3 a GPS receiver can be seen mounted on a tripod. A second tripod (off the picture to the right) provides a reference direction.

In general terms, the equipment notes the ephemeris data of each satellite in sight but, via a mobile telephone, uses information provided by a fixed base station some miles away to correct the data. The position of the base station on the National Grid is known to high precision.

Two pieces of surveyors' reflecting tape were stuck to the wall about 1 m apart and at the same horizontal level. Each piece of tape was marked with a cross and the two crosses were surveyed using a total station. The survey data was processed back in the office and a full report was delivered. This included the Grid Coordinates of the two points each specified to 1 mm precision relative to the National Grid reference origin off the Isles of Scilly.

These days the National Grid⁴ is *by definition* a transformation of the European Terrestrial Reference System 1989 (ETRS89) which, in turn, relates to the main European tectonic plate.

The report gave the orientation of the wall relative both to grid north and to true north. Defining true north is now almost as challenging as defining time but the indicated precision, about 3 arc-minutes, was sufficient for my purposes.

A precise estimate of latitude was bundled in at no extra charge. The values assumed in all subsequent calculations were:

$$\text{Latitude} = 52^\circ 12' 3'' \text{ and } \text{Azimuth} = 184^\circ 26'$$

The azimuth here is the azimuth of the outward normal to the dial measured clockwise round from true north.

Refining the Design

With the dimensions of the slate and the nodus height now known and the wall parameters determined too, I refined the first sketch. The revised design is shown in Fig. 4.

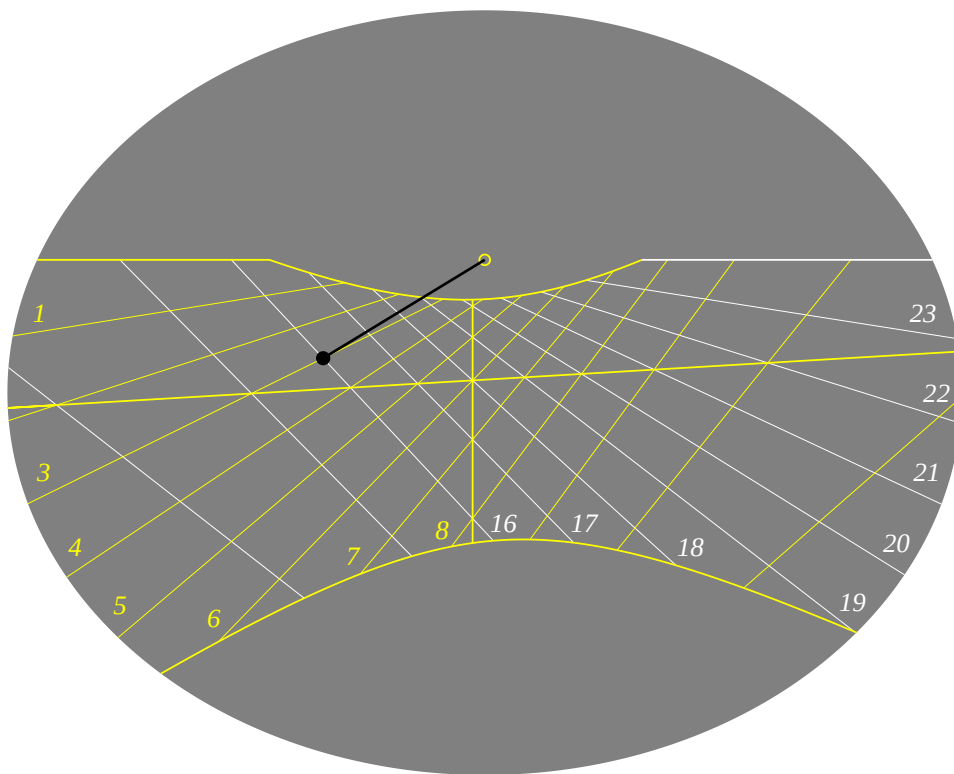


Fig. 4 — 3h Babylonian Hours, 16h Italian Hours and 09:30 French Hours

By this stage, I had discussed the design with Lida Cardozo Kindersley who suggested two immediate improvements.

First, she suggested dispensing with the inscription band. There was plenty of room for inscriptions above the horizon line and below the summer solstice curve. An important

additional benefit from this change was that there was more real estate on the west and east margins of the slate that could be used for the hour-lines.

Secondly, she suggested colour-coding the two families of hour-lines. We decided the Babylonian hour-lines should be gilded and the Italian hour-lines should be painted off-white.

These suggestions are incorporated in Fig. 4. Notice that the eastern end of the horizon line equates to Italian hour 24 and is therefore white. The labels on the Italian hour-lines were also changed to white.

There was considerable discussion about whether the Babylonian hour-lines should run across the Italian hour-lines or *vice versa* or whether the all lines should break at the crossing-points.

It was eventually decided that Babylonian hour-lines should go on top. It was also decided that the three constant-declination lines and the noon line should be gilded as in the first sketch.

Fig. 4 incorporates a synthetic shadow. Compare this with the shadow in Fig. 2. The shadow of the nodus falls on the Babylonian hour-line labelled 3 and it also falls on the Italian hour-line labelled 16.

It is therefore three hours after sunrise and 16 hours after sunset. Given that there are 24 hours in a day, there are $24 - 16 = 8$ hours *until* sunset. Hours that count downwards to sunset are sometimes called Co-Italian hours.

As will be explained later, the average of the two times, Babylonian hours and Italian hours, gives what will be referred to here as French hours, often called modern or common hours. This is the time that would be indicated on a normal sundial with a polar-oriented gnomon.

In Fig. 4, the average is $9\frac{1}{2}$, the average of 3 and 16, which corresponds to 09:30, two and a half hours before noon.

Further Refinements

By now, Lida Cardozo Kindersley had assigned one of her apprentices, Russell Purdham, to undertake the setting out and cutting. Several further refinements to the design were made.

First, Lida had more information about the proposed inscriptions and said that the design should be shifted upwards on the slate to make more space below the summer solstice curve to accommodate the inscription that was to go there.

Secondly, I noticed that, in my first sketch, the sub-nodus point was on the vertical centre-line of the slate, the minor axis. The design was shifted slightly to the right to ensure that the noon line falls on the minor axis of the ellipse instead.

Thirdly, Lida and Russell pointed out that the natural viewpoint would be several metres from the dial and the lines should be heavier. It was decided that all the gnomonic lines should be 5 mm wide.

Fourthly, Lida proposed that the top region of the dial should have a fiery sun. There would be space for a short inscription in the centre of the sun.

Finally, Russell solved a problem which is evident on the first sketch: one of the Babylonian hour-line labels is missing. To reduce clutter, it was decided early on to restrict the labelling of the Babylonian hours to the morning side of the noon line and the labelling of the Italian hours to the afternoon side. It was also thought that Babylonian hours are most useful in the mornings and Italian hours are most useful in the afternoons.

A more difficult decision was how to accommodate the label for Babylonian hour 2. This label is missing in Figs 1 and 4 because, in its natural position above the associated hour-line, it would fall across the left-hand end of the equinoctial line. Russell's solution was to have labels 1 to 5 below their respective hour-lines. This meant having 5 and 6 in the same cell but that doesn't spoil the appearance.

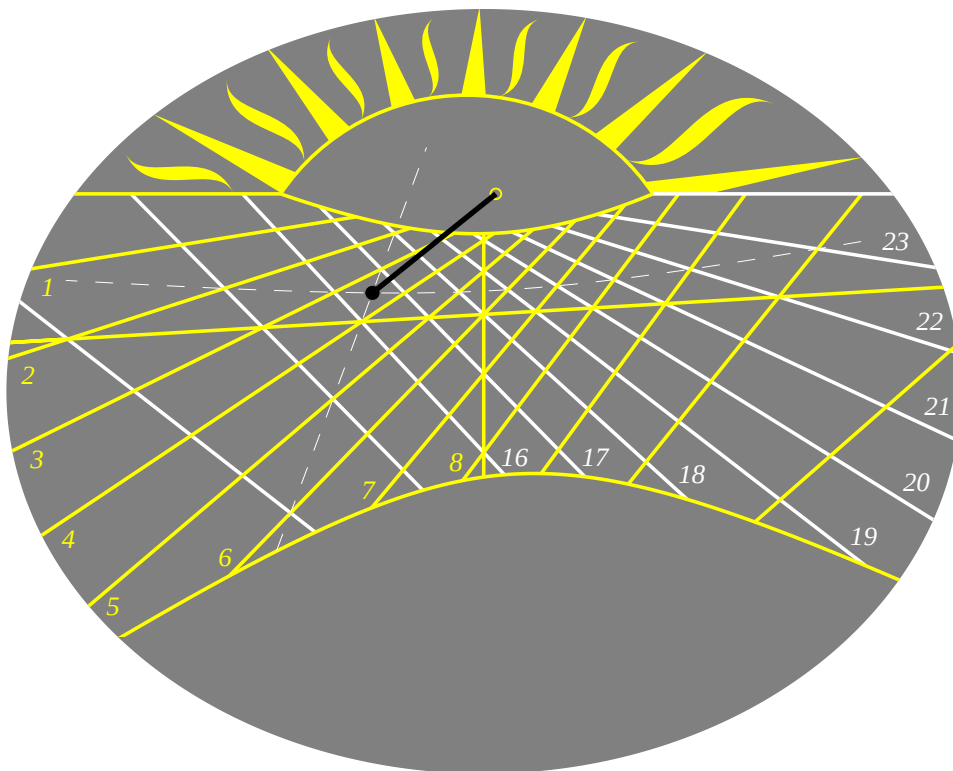


Fig. 5 — 3.5h Babylonian Hours, 16.5h Italian Hours and 10:00 French Hours

Fig. 5 shows my rendering of the design at this stage. The shadow of the nodus is shown half an hour later than it is in Fig. 4. The Babylonian and Italian hours are now 3.5h and 16.5h respectively. The average is 10 indicating that normal sundial time is 10:00.

The shadow of the nodus approximately follows a constant-declination curve during the course of a day and the relevant curve in the present case is shown as a broken line. This runs through a sequence of crossing-points in the criss-cross pattern. It will be shown later that for a constant-declination curve to run through the crossing-points in this way, the number of hours from sunrise to sunset must be an integer.

In Cambridge, the number of hours of daylight varies from fewer than 8 hours to more than 16 hours so there are nine declinations for which the constant-declination curve runs

through crossing-points. In the example shown, the number of hours of daylight is 11 and the declination is -5.78° .

A second broken line joins the crossing-points which correspond to 10h French hours. This is a straight line and would be the regular 10h hour-line on an ordinary sundial. As such, its upward extension intersects the upward extension of the noon line at what would be the root of the polar-oriented gnomon if this sundial had a gnomon.

Setting out

When a slate is being worked on, it is generally either flat on a table or almost vertical on an easel. Often, the slate is on a table for marking out and on an easel for cutting.

The traditional approach is to begin by establishing the major and minor axes of the ellipse. Ruler and compass constructions are not very helpful here and the usual method is to make a rubbing of the entire slate outline on tracing paper and fold the result in half both ways to establish the axes. The ends of these axes are transferred to the slate as light tick marks.



Fig. 6 — Russell Purdham setting out the Summer Solstice Curve

The next step was to mark the sub-nodus point whose coordinates (in millimetres) relative to the centre of the ellipse are (13.6, 223). The horizontal offset of 13.6 mm ensures that the noon line falls on the minor axes given a nodus height of 175 mm.

Russell and I then set out a grid of 100 mm squares, effectively making a large piece of graph paper with the sub-nodus point as the origin.

The hour-lines were set out first and, as they are straight lines, each can theoretically be specified by just two points. Nevertheless, I prepared a spreadsheet showing the coordinates of five points on most hour-lines. We drew best-fit straight lines through the points.

For the winter and summer solstice curves we used the end points of all appropriate hour-lines and several intermediate points as well. Using a flexible strip of wood and G-cramps we set out the winter and summer solstice curves as illustrated in Fig. 6.

A single line is not a suitable guide for a stone-cutter because the line disappears at the first cut with a chisel. The standard practice is to draw parallel lines on either side of the centre-line.

Although it is difficult to see in Fig. 6, each of the hour-lines is actually a triplet of lines with the outer lines 2.5 mm either side of the centre-lines. It is the outer lines which are used as guides for cutting.

Cutting

When all the gnomonic features had been set out, the slate was transferred to an easel and Russell embarked on the cutting. Fig. 7 shows the appearance of the slate after the equinoctial line, the summer solstice curve and most of the hour-lines had been cut.

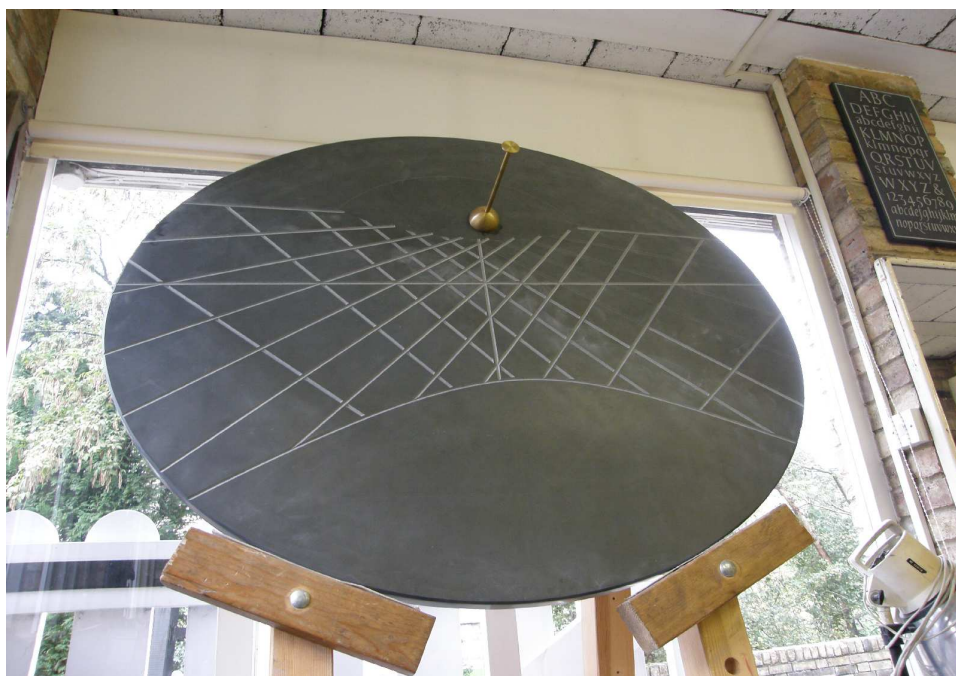


Fig. 7 — Most Hour-Lines now cut

The nodus has been included in the photograph to show where it was intended to go. It is held in place by three M6 bolts which are screwed in through a base-plate on the back of the slate. For most of the cutting stage the nodus was kept safely in a box.

Inscriptions

There is no sign of the sun or the inscriptions in Fig. 7 and the winter solstice curve is absent too.

About this stage, it was decided that the central part of the sun should be a stippled gilded region whose lower margin should form the winter solstice curve. The upper inscription

would be incorporated in this region too.

Fig. 8 shows the glorious sunburst complete and the upper inscription is complete too. Russell is working on the longer inscription below the summer solstice curve.

The upper inscription consists of the two Greek words, KAIPON ΓΝΩΘΙ, attributed to Pittacus, one of the Wise Men of Greece. These words translate as ‘know the time’ or ‘recognise an opportunity’. This inscription also appears on a sundial on the south side of Ely Cathedral.



Fig. 8 — Russell Purdham cutting the Latin Inscription

In the space below the summer solstice curve there is a Latin inscription, *Collegio suo lactarius Eboracensis me dono dedit*, which is a reference to the benefactor: To his college the Yorkshire dairyman gave me as a gift.

By now the identity of the benefactor had been made public. He is Jim Dickinson, a Selwyn College alumnus, and the Latin refers to just one of his professional interests.

Later, two further inscriptions were cut into the rim of the dial in the thickness of the slate itself. These are explanations: BABYLONIAN HOURS SINCE SUNRISE and ITALIAN HOURS SINCE SUNSET.

The inscriptions were almost the last items to be cut. The labels on the hour-lines are the major outstanding items in Fig. 8.

Finishing

After concluding the cutting, the slate was washed and gold leaf was applied to almost all the dial furniture except for those elements that relate to the Babylonian hours: the

hour-lines, the labels and the instruction in the rim. The nodus and its support were gilded too.

The slate was now almost ready for fixing to the wall. The traditional way of fixing slate plaques and sundials is to drill three holes a short distance into the back of the slate and glue in three pins which are used as studding.

The holes are 15 mm in diameter and were drilled 20 mm into the thickness of the slate. The fixing pins are 120 mm long so 100 mm protrudes.

The slate was taken to Selwyn where the scaffolding that had been used by the surveyors had been erected again. A template was used to position the three holes in the wall. The holes were drilled and the slate was lifted into position using a good deal of muscle power. See Fig. 9.



Fig. 9 — Lifting the Finished Dial into Position

The slate was kept in transparent protective covering during this stage and the nodus was still in its box. Assuming the survey data was correct and all the calculations and setting out was properly carried out, there should be no need to do anything more than to ensure that the horizon line is horizontal and the face of the slate is vertical.

Some walls have such undulating surfaces that a large slate can be rocked to and fro. In such circumstances, if the sun is shining, I can cant the dial slightly so that it faces in the intended direction.

The 19th century brickwork at Selwyn was remarkably true and no such adjustments were necessary. The sun was indeed shining and I could satisfy myself that there was nothing horribly wrong. This is a nervous moment for the diallist!

The dial was lifted off the wall ready for the permanent fixing. The protective cover was taken off the slate and the nodus was bolted into place. Russell wore white gloves for this task. Plastic padding was squeezed into the holes in the wall, the slate was offered up again and I had a few brief minutes to make final checks. The plastic padding goes off very quickly.

The scaffolding was taken down and the sun continued to shine for just a few minutes. The finished dial was at last in its intended position. See Fig. 10.



Fig. 10 — The Finished Dial

Some Notes the Criss-Cross Pattern

The criss-cross pattern formed by the intersection of the two families of hour-lines has some interesting and easy-to-understand numerical properties.

Consider what happens at an equinox when there are 12 hours of day and 12 hours of night. Expressed in French hours (normal local sun time), sunrise is at 6h and sunset is at 18h. Expressed in Babylonian hours, sunrise is at 0h and, expressed in Italian hours, sunrise is at 12h because it is 12 hours since sunset.

Let's represent these three sunrise times as:

$$F = 6 \quad B = 0 \quad I = 12$$

where F refers to French hours, B refers to Babylonian hours and I refers to Italian hours. Babylonian hours are six hours behind French hours and Italian hours are six hours ahead of French hours. These differences persist all day and can be expressed as:

$$B = F - 6 \quad I = F + 6$$

At noon when $F = 12$ it is clear that $B = 6$ and $I = 18$.

Whenever F is an integer, B and I are also integers so Babylonian and Italian hour-lines intersect at hourly intervals along the equinoctial line, in particular at $F = 12$ where the noon line contributes to the intersection.

During the summer months the days are longer than 12 hours. There is extra daylight before 6h and extra daylight after 18h (both times being French hours). Let d be the number of extra hours of daylight, at each end of the day. The relationships become:

$$B = F - 6 + d \quad I = F + 6 - d$$

Now, at noon when $F = 12$ the Babylonian hours time is later than 6h by an amount d , because the sun is rising d hours earlier than at an equinox. Likewise, the Italian hours time is earlier than 18h by an amount d because the sun is setting d hours later. Note that the relationships are also valid in winter but d is then negative.

Summing the two left-hand sides and the two right-hand sides gives:

$$B + I = 2F$$

This explains why the time in French hours is the average of the times in Babylonian and Italian hours: $F = (B + I)/2$.

Instead of summing, one can subtract:

$$B - I = -12 + 2d$$

Now add 24 to each side and rearrange:

$$B - I + 24 = 12 + 2d \quad \text{so} \quad B + (24 - I) = D \quad \text{or} \quad B + C = D$$

Here C refers to Co-Italian hours, $(24 - I)$, and D refers to the length of the day from sunrise to sunset. This is 12 hours plus two lots of extra daylight d . It is self-evident that $B + C$ (the time since sunrise plus the time until sunset) is D (the length of day).

The relationship $B + (24 - I) = D$ is important. If any two of the three variables is an integer then the third is also an integer. In particular, if the number of hours of daylight D is an integer then whenever either B or I is an integer the other must also be an integer so the shadow of the nodus will pass through a sequence of crossing-points.

When D is an integer it is possible to read French hours almost directly. The shadow of the nodus runs through a sequence of crossing-points which are at one-hour intervals. In Fig. 4 it is immediately clear that the time is $2\frac{1}{2}$ hours before noon and in Fig. 5 it is equally clear that the time is 2 hours before noon.

The extra daylight d depends on the latitude ϕ and solar declination δ . These three entities are related by:

$$\sin d = \tan \phi \tan \delta$$

Here d is expressed as an hour-angle so, to determine the declination required for the length of the day to be 11 hours, one sets $d = -7.5^\circ$. This takes 7.5° off each end of a 12-hour day, a total of 15° , which corresponds to one hour.

Taking the latitude of Selwyn as $52^\circ 12' 3''$:

$$\tan \delta = \frac{\sin d}{\tan \phi} = \frac{\sin(-7.5)}{\tan(52^\circ 12' 3'')} = -0.1012$$

Accordingly, $\delta = -5.78^\circ$. This is the declination of the constant-declination curve drawn as a broken line in Fig. 5.

In Cambridge, the number of hours of daylight D varies from fewer than 8 hours to more than 16 hours. There are therefore nine possible integer values of D and hence nine declinations for which the constant-declination curve runs through crossing-points. In the example just given, the number of hours of daylight is 11 and the declination is -5.78° .

Using Hour-Angles and Declinations for Setting out

The nine declinations for which the associated constant-declination curve runs through a sequence of crossing-points are given in the table shown as Fig. 11.

D	d	δ
8	-2.0	-21.20
9	-1.5	-16.53
10	-1.0	-11.35
11	-0.5	-5.78
12	0.0	0.00
13	0.5	5.78
14	1.0	11.35
15	1.5	16.53
16	2.0	21.20

Fig. 11 — Nine Special Declinations

The nine integer lengths of day D range from 8 to 16. For each, d is the amount of extra daylight (relative to a 12-hour equinoctial day) at each end of the day, so $d = (D - 12)/2$. For each d the associated declination δ is determined as just illustrated.

Anyone wishing to see the shadow of the nodus running across a sequence of crossing-points should arrange to visit on a day when the solar declination is one of the values in the table. There are 18 such occasions each year.

These special declinations give rise to one possible method of setting out Babylonian and Italian hour-lines. The scheme is illustrated in Fig. 12.

The procedure begins by choosing a sub-nodus point and a nodus height. The sub-nodus point is again shown as a yellow circle. One then determines where a hypothetical polar-oriented style through the nodus would intersect the dial. This root-of-the-gnomon point is shown as a blue cross. It is just off the top of the slate. The line from the cross to the circle is the sub-style.

Next, draw a set of common hour-lines radiating from the root of the gnomon. The hour-lines from 8h to 17h are shown in the figure but only those from 10h to 15h are labelled.

Then, draw the constant-declination curves for the nine special declinations given in Fig. 11. All nine constant-declination lines are shown in the figure but only those for $D = 12$, $D = 13$ and $D = 16$ are labelled.

The French hour-lines and the constant-declination curves are both shown in blue and together provide a means of plotting points in terms of their hour-angles and declinations. The only novelty is that the lines of constant-declination are identified by their associated D -values rather than by their declinations.

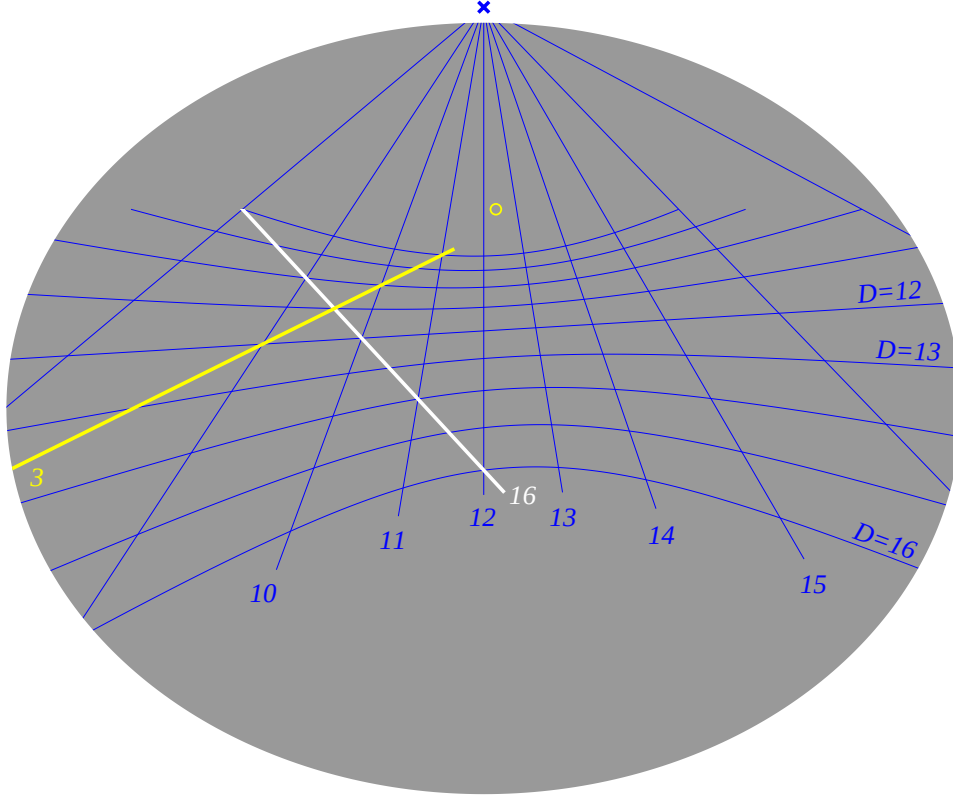


Fig. 12 — Setting out Babylonian and Italian Hour-Lines

Every Babylonian and Italian hour-line runs through a set of crossing-points and two sample lines are shown, 3h Babylonian and 16h Italian. The Babylonian hour-line passes through three intersection points and the Italian hour-line passes through five.

From the relationships given above it is easy to derive two more:

$$D = 2(12 + B - F) \quad \text{and} \quad D = 2(12 - I + F)$$

To plot the 3h Babylonian hour-line, use the left-hand relationship and set $B = 3$ giving $D = 2(15 - F)$. From this, tabulate a set of (F, D) pairs as $(9, 12)$, $(10, 10)$ and $(11, 8)$. Mark these intersections and join them up.

To plot the 16h Italian hour-line, use the right-hand relationship and set $I = 16$ giving $D = 2(F - 4)$. From this, tabulate a set of (F, D) pairs as $(8, 8)$, $(9, 10)$, $(10, 12)$, $(11, 14)$ and $(12, 16)$. Mark these intersections and join them up.

The two hour-lines intersect at 09:30 French hours. The point of intersection is where the shadow of the nodus falls in Fig. 4. There is a case for marking in French half-hour lines but they are omitted from Fig. 12 to reduce clutter.

The winter and summer solstice lines have not been drawn but they are just as on any sundial which has these features.

In summary: anyone who can set out ordinary hour-lines and constant-declination curves can readily set out Babylonian hour-lines and Italian hour-lines. Note that the method described applies to a dial of arbitrary orientation and there is not even a requirement for it to have a plane surface.

The Horizon Line

Although unlabelled, the 0h Babylonian hour-line and the 24h Italian hour-line both feature on the Selwyn Dial. These lines coincide with projections of parts of the eastern and western horizon respectively.

The noted Italian *gnomonista*, Gianni Ferrari, has pointed out⁵ that since any straight line can be specified by two points, it is unnecessary to draw all the constant-declination curves in Fig. 12. All that is required is to note where the French hour-lines (and half-hour lines) intersect the equinoctial line and the horizon line.

To plot the 16h Italian hour-line, note where the $F = 10$ line intersects the equinoctial line and where the $F = 8$ line intersects the horizon line. The $F = 10$ stems from the relationship $F = I - 6$ which applies at the equinoxes. The $F = 8$ stems from the relationship $F = (B + I)/2$, given that on the horizon line $B = 0$.

To plot the 3h Babylonian hour-line, note where the $F = 9$ line intersects the equinoctial line and where the $F = 13\frac{1}{2}$ line intersects the horizon line. The $F = 9$ stems from the relationship $F = B + 6$ which applies at the equinoxes. The $F = 13\frac{1}{2}$ stems from the relationship $F = (B + I)/2$, given that on the horizon line $I = 24$.

Gianni Ferrari also explained how this method has to be adapted slightly for horizontal dials where the horizon line is at infinity...

The 16h Italian hour-line again intersects the equinoctial line at $F = 10$ and, in some sense, again intersects the horizon line at $F = 8$ but, since the horizon line is at infinity, this requires the hour-line to be drawn *parallel* to the $F = 8$ line.

The 3h Babylonian hour-line again intersects the equinoctial line at $F = 9$ and, in some sense, again intersects the horizon line at $F = 13\frac{1}{2}$ which requires the hour-line to be drawn *parallel* to the $F = 13\frac{1}{2}$ line.

These procedures are theoretically sound but have limitations in practice. The relevant intersection points may be off the dial. At Selwyn, the 11h Babylonian hour-line intersects both the equinoctial line and the horizon line off the dial.

It has already been noted that the French hour-lines radiate from a point which is also off the dial. In practice, the coordinates of several points on each hour-line were computed and marked on the slate. These points included intersections with the winter and summer solstice curves where appropriate.

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