Incoop: MapReduce for Incremental Computations

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ABSTRACT

Many online data sets grow incrementally over time as new entries are slowly added and existing entries are deleted or modified. Taking advantage of this incrementality, systems for incremental bulk data processing, such as Google’s Percolator, can achieve efficient updates. This efficiency, however, comes at the price of losing compatibility with the simple programming models offered by non-incremental systems, e.g., MapReduce, and more importantly, requires the programmer to implement application-specific dynamic/incremental algorithms, ultimately increasing algorithmic complexity.

In this paper, we describe the architecture, implementation, and evaluation of a generic MapReduce framework, named Incoop, for incremental computations. Incoop detects changes to the inputs and enables the automatic update of the outputs by employing an efficient, fine-grained result re-use mechanism. To achieve efficiency without sacrificing transparency, we adopt recent advances in the area of programming languages to identify systematically the shortcomings of task-level memoization approaches, and address them using several novel techniques such as a storage system to store the input of consecutive runs, a contraction phase that make the incremental computation of the reduce tasks more efficient, and a scheduling algorithm for Hadoop that is aware of the location of previously computed results.

We implemented Incoop by extending the Hadoop framework, and evaluated it with a variety of applications, including two case studies of higher-level services: incremental query (based on Pig) and log processing systems. Our results show significant performance improvements without changing a single line of application code.

1. INTRODUCTION

As computer systems produce and collect increasing amounts of data, analyzing it becomes an integral part of improving the services provided by Internet companies. In this context, the MapReduce framework offers techniques for convenient, distributed processing of data by enabling a simple programming model that eliminates the burden of implementing a complex logic or infrastructure for parallelization, data transfer, scalability, fault tolerance and scheduling.

An important property of the workloads processed by MapReduce applications is that they are often incremental by nature; i.e., MapReduce jobs often run repeatedly with small changes in their input. For instance, search engines will periodically crawl the Web and perform various computations on this input, such as computing a Web index or the PageRank metric, often with very small modifications, e.g., at a ratio of ten to thousand, in the overall input [17, 12].

This incremental nature of data suggests that performing large-scale computations incrementally can improve efficiency dramatically. Broadly speaking there are two approaches to achieve such efficient incremental updates. The first approach would be to devise systems that provide the programmer with facilities to store and use state across successive runs so that only computations that are affected by the changes to the input would need to be executed. This is precisely the strategy taken by major Internet companies who developed systems like Percolator [17] or CBP [12]. This approach, however, requires adopting a new programming model and a new API that differs from the one used by MapReduce. These new programming APIs also require the programmer to devise a way to process updates efficiently, which can increase algorithmic and software complexity. For example, research in the algorithms community on algorithms for dynamically or incrementally changing data show that such algorithms can be very complex even for problems that are relatively straightforward in the non-incremental case [6, 8]. The second approach would be to develop systems that can reuse the results of prior computation transparently. This approach would shift the complexity of incremental processing from the programmer to the processing system, essentially keeping the spirit of high-level models such as MapReduce. A few proposals have taken this approach, e.g., DryadInc [18], and Nectar [9], in the context of the Dryad system by providing techniques for task-level or LINQ expression-level memoization.

In this paper we present a system called Incoop, which allows existing MapReduce programs, not designed for incremental processing, to execute transparently in an incremental manner. In Incoop, computations can respond automatically and efficiently to modifications to their input data by reusing intermediate results from previous computations, and incrementally updating the output according to the changes in the input. To achieve efficiency, Incoop relies...
on memoization but goes beyond the straightforward task-level application of this technique by performing a stable partitioning of the input and by reducing the granularity of tasks to maximize result re-use. Furthermore, Incoop employs affinity-based scheduling techniques to further improve the performance of the system.

In particular, the design of Incoop contains the following new techniques that we incorporated into the Hadoop MapReduce framework, and which we show to be instrumental in achieving efficient incremental computations.

- **Incremental HDFS.** Instead of relying on HDFS to store the input to MapReduce jobs, we devise a file system called *Inc-HDFS* that provides mechanisms to identify similarities in the input data of consecutive job runs. The idea is to split the input into chunks whose boundaries depend on the file contents so that small changes to input do not change all chunk boundaries. Inc-HDFS therefore partitions the input in a way that maximizes the opportunities for reusing results from previous computations, while preserving compatibility with HDFS, by offering the same interface and semantics.

- **Contraction phase.** We propose techniques for controlling the granularity of tasks so that large tasks can be divided into smaller subtasks that can be re-used even when the large tasks cannot. This is particularly challenging in Reduce tasks, whose granularity depends solely on their input. Our solution is to introduce a new Contraction phase that leverages Combiner functions, normally used to reduce network traffic by anticipating a small part of the processing done by Reducer tasks, to control the granularity of the Reduce tasks.

- **Memoization-aware scheduler.** To improve effectiveness of memoization, we propose an affinity-based scheduler that uses a work stealing algorithm to minimize the amount of data movement across machines. Our new scheduler strikes a balance between exploiting the locality of previously computed results and executing tasks on any available machine to prevent straggling effects.

- **Use cases.** We employ Incoop to demonstrate two important use cases of incremental processing: *incremental log processing*, where we use Incoop to build a framework to incrementally process logs as more entries are added to them; and *incremental query processing*, where we layer the Pig framework on top of Incoop to enable relational query processing on continuously arriving data.

We implemented Incoop and evaluated it using five MapReduce applications and the two use cases. Our results show that we achieve significant performance gains, while incurring only a modest penalty during runs that cannot take advantage of memoizing of previous results, namely the initial run of a particular job. The results also show the effectiveness of the individual techniques we propose.

The rest of this paper is organized as follows. Section 2 presents a system overview of Incoop. The system design is detailed in Sections 3, 4, and 5. We present an analysis in Section 6, our system implementation and its experimental evaluation in Section 7. Finally, related work and conclusions are discussed in Section 8 and Section 9, respectively. Due to space restrictions, we cover the two case studies (Appx A) and the proofs for the analytic performance study (Appx B) in appendices.

## 2. SYSTEM OVERVIEW

This section presents the goals, basic approach, and main challenges the underlie the design of Incoop.

### 2.1 Goals

Our goal is to devise a system for large-scale data processing that is able to realize the performance benefits of incremental computations, while keeping the application complexity and development effort low. Specifically, we aim to achieve transparency and efficiency.

- **Transparency.** A transparent solution can be applied to existing bulk data processing programs without changing them. This (i) makes the approach automatically applicable to all existing programs while preserving the full generality of the approach, and (ii) requires no additional effort from the programmer to devise and implement an efficient incremental update algorithm.

- **Efficiency.** One of the lessons to take away from the work on programming language support for the automatic incrementalization of single-machine programs [2] is that this transparent incremental processing can often be asymptotically more efficient (often by a linear factor) than complete, from-scratch recomputation. At the scale of the bulk data processing jobs that run in today’s data centers, these asymptotic improvements can translate to huge speedups and cluster utilization savings. We aim to realize such speedups in practice.

Even though it would be possible to devise solutions that work with various types of data processing systems, in this paper, we target the MapReduce model, which has emerged as a de facto standard for programming bulk data processing jobs. The remainder of this section presents an overview of the challenges in the design of Incoop, an extension of Hadoop that provides transparent incremental computation of bulk data processing tasks.

Our design adapts the principles of self-adjusting computation (§8) to the MapReduce paradigm and the Hadoop framework. The idea of self-adjusting computations is to keep track of the dependencies between the inputs and outputs of different parts of a single-machine computation, and only rebuild the parts of the computation affected by changes. In particular, a computation graph records the dependencies between data and between (sub)computations. Nodes of the computation graph represent subcomputations and edges between nodes represent update-use relationships, i.e., dependencies, between computations: there is an edge from one node to another if the latter subcomputation uses some data generated by the former. To change the input dynamically, the programmer indicates which parts of the input were added, deleted, and modified (using a special-purpose interface) and the computation graph is used to determine the subcomputations (nodes) that are affected by these changes. The affected subcomputations are then executed, which can recursively make new subcomputations
affected, while subcomputations that remain unaffected are re-used by a form of computation memoization.

Although self-adjusting computation offers a general-purpose framework for developing computations that can perform incremental updates efficiently, it has not been applied to a distributed setting. In addition, the approach does not support transparent application, and its efficiency critically depends on certain properties of the computation. In the rest of this section, we briefly outline these challenges and how we overcome them.

2.2 Challenges: Transparency

Self-adjusting computation traditionally requires the programmer to annotate programs with specific language primitives. These primitives help the compiler and the run-time system identify dependencies between data and computations. Similarly the interface for making changes to the input help identify the “edits” to the input, i.e., how the input changes. To adapt this approach to the MapReduce model without having to require programmer annotations, we need to address some important questions.

Input changes. Our goal of transparency, coupled with the fact that the file system used to store inputs to MapReduce computations (HDFS) is an append-only file system, makes it impossible to convey changes to the input (other than appending new data). To maintain a backwards-compatible interface, but still allow for incremental processing, we allow for both the inputs and the outputs of consecutive runs to be stored in separate HDFS files. Alternatively, we also allow for new data to be periodically appended to the input, e.g., when repeatedly processing a log that grows throughout the system lifetime (Appx A.1). Throughout the description we will consider the first approach, since it is more general.

Programmer annotations. To eliminate annotations from program code, we specialize self-adjusting computation techniques to the MapReduce framework by exploiting the structure of MapReduce computations. Specifically, we exploit the fact that the framework implicitly keeps track of the dependencies between the various tasks, which form a natural unit of sub-computation.

Based on these ideas, we arrive at a simple design that we use as a starting point, which is depicted in Figure 1, and can be described at a high-level as follows. We add a memoization server that aids in locating the results of parts of previous computations. The responsibility of this server is to store a mapping from the input of a previously run task to the location of the respective output. During a run, whenever a task completes, its output is stored persistently, and a mapping from the input to the location of the output is stored in the memoization server. Then, whenever a task runs, the memoization server is queried to check if the inputs to the task match those of a previous run of the computation. If so, we re-use the outputs that were kept from that previous run. This way only the parts of the computation affected by inputs changes are re-computed. Finally, we periodically identify and purge old entries from the memoization server to ensure that storage doesn’t grow without bounds.

2.3 Challenges: Efficiency

To achieve efficient dynamic updates, we must ensure that the MapReduce computations remain stable under small changes to their input. Specifically, we define stability as follows. Consider performing MapReduce computations with inputs $I$ and $I'$ and consider the set of tasks that are executed, denoted $T$ and $T'$ respectively. We say that a task $t \in T'$ is not matched if $t \not\in T$, i.e., the task that is performed with the second inputs $I'$ is not performed with the same input. We say that a MapReduce computation is stable if the time required to executed the unmatched tasks is small, ideally, sub-linear in the size of the input. More informally, a MapReduce computation is stable if when executed with similar inputs, the set of tasks that are executed are also similar, i.e., many tasks are repeated.

Achieving stability in MapReduce requires overcoming several important challenges: a) making a small change to the input can change the input to many tasks ultimately leading to a large number of unmatched tasks; b) even if a small number of tasks are affected, the tasks themselves can require a large time to execute. To solve these problems, we propose techniques for (1) performing a stable partitioning of the input; (2) controlling the granularity and stability of the Map and Reduce tasks; (3) finding efficient scheduling mechanisms for identifying the affinity of tasks to machines to maximize benefits of result re-use. We briefly summarize our design decisions below.

Stable Input Partitions. To see why the standard MapReduce approach to input partitioning leads to unstable computations, consider inserting a single record in the beginning of an input file. Since the input is partitioned into fixed-sized chunks, this small change will shift each partition point by one record, effectively changing the input to each map task. In general, when the record is inserted at some position, all chunks that follow that position will have to shift by one, and thus on average nearly half of all tasks will be unmatched. The problem only gets more complicated as we allow more complex changes, where for example the order of records may be permuted; such changes can be common for instance, if a crawler uses a depth-first strategy to crawl the web, and a single link changing can move an entire subtree’s position in the input file. One possible solution to this problem would be to compute the differences between the two inputs files and update somehow the computation by using this difference directly. This would, however, require running a polynomial-time algorithm, (e.g., an edit-distance algorithm) to find the difference.

Our solution is to use a stable partitioning technique that enables maximal overlap between the set of data chunks created with similar inputs. Maximizing the overlap be-
tween data-chunks in turn enables maximizing the number of matched Map tasks. To support stable partitioning we propose a file system, called Inc-HDFS that we describe in Section 3.

**Granularity control.** We maximize the overlap between Map tasks by using a stable partitioning technique for creating the input data chunks. The input to the Reduce tasks, however, is directly determined by the outputs of the Map tasks, because the key-value pairs with the same key are processed together by the same Reduce task. This raises a problem if, for example, a single new key-value pair is added to a Reduce task that processes a large number of values, deeming these large tasks to be unmatched. Furthermore, even if we found a way of dividing large Reduce tasks into multiple pieces, this would not solve the problem in case such tasks depended on each other, e.g., if the input of a task depended on the output of the previous. Thus, we need a way to reduce not only the task size but also eliminate potentially long (often linear-size) dependencies between parts of the Reduce tasks. In other words, we need to control granularity without increasing the number of unmatched tasks.

We solve this problem by performing an additional Contraction phase where Reduce tasks are combined hierarchically in a tree-like fashion; this both controls the task size and ensures that no long dependencies between tasks arise (all paths in the tree will be of logarithmic length). Section 4 describes our proposed approach.

**Scheduling.** To enable efficient re-use of matched computations, it is important to schedule a task on the machine that holds the memoized data it uses. We achieve this by extending the scheduling algorithm used by Hadoop with a notion of **affinity**, that allows the scheduler to take into account “affinities” between machines and tasks by keeping a record of which nodes have executed which tasks. This simple approach will minimize the movement of memoized intermediate results by ensuring that data movement is localized to the node. However, this solution results in an overall degradation of job performance by introducing stragglers [20], since a strict affinity of tasks results in deterministic scheduling and prevents a lightly loaded node to steal work from a slow node’s task queue. We therefore propose a hybrid scheduling policy that strikes a balance between work-stealing and affinity to the memoized results. The detailed description of the modified scheduler is covered in Section 5.

The next three sections describe how the design of Incoop addresses the aforementioned challenges.

**3. INCREMENTAL HDFS**

We propose Incremental HDFS (Inc-HDFS) as a distributed file system that assists Incoop in performing incremental computations efficiently. Inc-HDFS extends the Hadoop distributed file system (HDFS) to enable stable partitioning of the input via content-based chunking, which was introduced in LBFS [14] for data deduplication. At a high-level, content-based chunking defines chunk boundaries only based on the local input contents, instead of static chunk sizes. As a result, even with incremental changes, the chunking changes by a small amount and hence thus the inputs to MapReduce tasks remain stable, i.e., similar to those of the previous run. Figure 2 illustrates a comparison of the chunking strategies between standard HDFS and Inc-HDFS.

To perform content-based chunking we scan the entire file using a fixed-width sliding window. For each file position, we read the window contents, and compute a Rabin fingerprint. If the fingerprinted content matches a certain pattern, which we term a **marker**, we place a chunk boundary at that position. A limitation of this approach is that it may create chunks that are too small or too large, given that markers will not be evenly spaced, and that the chunk size depends solely on the input and only the average size can be controlled by the system. To address this, we add a constraint of having a minimum and maximum chunk size. Thus, after we find a marker \( m_i \) at position \( p_i \), we skip a fixed offset \( O \) in the input sequence and continue to search for a marker starting at position \( p_i + O \). In addition, we bound the chunk length by setting a marker after \( M \) content bytes when no marker was found before that. Despite the possibility of affecting stability by either missing important markers due to skipping the initial offset, or consecutively use the maximum length in an unaligned manner, we found this scheme to work well in practice, in that such occurrences were very rare and had a minor impact on performance.

Chunking could be performed either during the creation of the input or when the input is read by the Map task. We choose the former approach for two main reasons. First, the additional cost for chunking can be amortized when the chunking and the actual generation of the input data can be performed in parallel. This is particularly advantageous when the process for input data generation is not limited by the storage throughput. The second reason is that when the input is first written to HDFS, it is already present in the main memory of the node that writes the input, and hence, this node can perform the chunking without additional accesses to the data.

In order to leverage the common availability of multicore during the chunking process, we parallelized the search for markers in the input data. Our implementation uses multiple threads that each search for markers in the input starting at different positions. The markers found can not be used directly to define the chunk boundaries, since some of them might be skipped in the sequential marker search. Instead,
we collect the markers in a list, and iterate over the list to determine the markers that are skipped and those that define the actual chunk boundaries. Our experimental evaluation (§7.4) highlights the importance of these optimizations in keeping the performance of Inc-HDFS very close to its original HDFS counterpart.

4. INCREMENTAL MAPREDUCE

We describe the incremental MapReduce part of the infrastructure by separately considering how Map and Reduce tasks handle incremental inputs.

Incremental Map. Given that Inc-HDFS already provides the necessary control over the alignment and granularity of the input parts that are provided to Map tasks, the job of the incremental Map tasks becomes simplified, since they can implement task-level memoization without having to worry about finding finer-grained, misaligned, or misplaced opportunities for reusing previous results. Specifically, after a Map task runs, we store its results persistently (instead of discarding it after task execution) and insert a corresponding reference to the result at the memoization server. Instances of incremental Map tasks take advantage of previously stored results by querying the memoization server. If they find that the result has already been computed, they fetch the result from the location of their memoized output, and conclude. This is illustrated in Figure 3 where part (a) describes the first run of an Incmap job, and part (b) describe what happens in a subsequent run where Split-2 is modified (hence replaced by Split-4) and the Map tasks for splits 1 and 3 need not be re-executed.

Incremental Reduce. The Reduce function processes the output of the Map function grouped by the keys of the generated key-value pairs. More precisely, for a subset of all keys, each Reduce retrieves the key-value pairs generated by all Map tasks and applies the Reduce function. To ensure efficient memoization of Reduce tasks, we perform memoization at two levels: first as a coarse-grained memoization of entire Reduce tasks, and second as a fine-grained memoization of novel Contraction tasks as described below.

As with Map tasks, we remember the results of a Reduce task by storing persistently and locally their result and by inserting a mapping from a collision-resistant hash of the input to the location of the output in the memoization server. Since a Reduce task receives input from all Map tasks, the key stored in the memoization server consists of the hashes of the outputs from all Map task that collectively form the input to the Reduce task. When executing a Reduce task, instead of immediately copying the output from the Map tasks, the Reduce task consults Map tasks for their respective hashes to determine if the Reduce task has already been computed in previous run. If so, that output is directly fetched from the location stored in the memoization server, which avoids the re-execution of that task.

This task-level memoization has a crucial limitation. Since Reduce tasks can be large, when not re-used due to a change in a small subset of its input, this can result in inefficient incremental updates. Furthermore, the larger the tasks, the more likely that its input will include some changed data and thus the less likely that it will be re-used. Since each Reduce processes all values that are produced for a given key and since the number of such values only depends on the computation and its input, we cannot control the size of the input to a reduce by partitioning the input. This ultimately hinders stability. We therefore need a way to decrease the granularity of Reduce tasks. This, however, is challenging because we must avoid creating long chain of dependencies between the smaller tasks — such dependencies will force the execution of many subtasks ultimately destroying the initial goal.

To reduce the granularity of Reduce tasks effectively, we propose a new Contraction Phase, which is run by Reduce tasks. To this end, we take advantage of a feature of the original MapReduce and the Hadoop frameworks [7] that was designed for a completely different purpose: Combiners. Combiners are meant to save bandwidth by offloading part of the computation performed by the reducer to the Map task. With this mechanism, the programmer of the task specifies a separate Combiner function, which is executed on the machine that runs the Map task, and pre-processes various (key,value) pairs, merging them into a smaller number of pairs. The signature of the combiner function has the same input and output types (a sequence of (key,value) pairs) and, very often, Combiners and Reducers perform very similar work.

We use Combiners to break up larger Reduce tasks into many applications of the Combine function, which allows us to perform memoization at a much finer granularity. More precisely, we split the Reduce input into chunks, and apply the Combiner function to each chunk. Then we again form chunks from the Combine result and recursively apply the Combiner function to these new chunks. The data size gets smaller in each iteration, and finally, we apply the Reduce function to the output of the last level of Combiners. This approach enables us to memoize the results of the Combiners and, when the input to the Reducer is changed, only a subset of the Combiners have to be re-executed rather than a full Reduce task.

This new usage of Combiners is compatible with the original Combiner interface, since both the input and the output of Combiners is a set of tuples that can be passed to the Reducer task. However, semantically, Combiners were only meant to optionally run once, and therefore the correctness of the computation it performs only is only required to be maintained across a single Combiner invocation, that is:

\[ R \circ C \circ M = R \circ M \]

where \( R \), \( C \), and \( M \) represent the Reducer, Combiner and Map function, respectively. Our new usage of Combiner function require a slightly different requirement:

\[ R \circ C^n \circ M = R \circ M, \forall n > 0 \]
Even though it is theoretically possible to write a Combiner that meets the original requirement but not the new one, in practice, all of the Combiner functions we found until today obeyed the new requirement.

Stability of the Contraction Phase. An important design question is how to partition the input to the Contraction phase into chunks that are processed by different Combiners. In this case, the same issue that arose at the Map phase needs to be handled: if a part of the input to the Contraction phase is removed or a new part is added, then partitioning the input by offset would undermine the stability of the dependence graph, since that small change could cause a large recomputation. This problem is illustrated in Figure 4, which shows two consecutive runs of the same Reducer, where a new map task (#2) is added to the set of map tasks that produce values associated with the key being processed by this Reducer. In this case, a simple partitioning of the input, e.g., into groups with a fixed number of input files, would cause all groups of files to become different from one run to the next, due to the insertion of one new file near the beginning of the sequence.

To solve this, we again rely on content-based chunking, and apply it to every level of the tree of combiners that forms the Contraction phase. The way we perform content-based chunking in the Contraction phase differs slightly from the approach we took in Inc-HDFS, due to both efficiency and simplicity reasons. In particular, given that the Hadoop framework splits the input to the contraction phase into multiple files coming from different Mappers, we restrict chunk boundaries to be at file boundaries, i.e., chunking can only group entire files. This way we leverage the existing partitioning of the input, which simplifies the implementation and avoids re-processing this input: we use the hash of each input file to determine if a marker is present, i.e., if that input file should be the last of a set of files that is given to a single Combiner. In particular, we test if that hash modulo a pre-determined integer $M$ is equal to a constant $k < M$. This way the input file contents do not need to be scanned to partition this input.

Figure 4 also illustrates how content-based chunking obviates the alignment problem. In this example, the content-based marker that delimits the boundaries between groups of input files is present in outputs #5, 7, and 14, but not the remaining ones. Therefore, inserting a new map output will change the first group of inputs but none of the remaining ones. In this figure we can also see how this change propagates to the final output. In particular, this change will lead to executing a new Combiner (labelled 1-2-3-5), and the final Reducer. The results for all of the remaining Combiners are reused without needing to re-execute them. This technique is then repeated across all levels of the tree.

5. MEMOIZATION-AWARE SCHEDULER

The Hadoop scheduler assigns Map and Reduce tasks to nodes for efficient execution, taking into account machine availability, cluster topology, and the locality of input data. The Hadoop scheduler, however, is not well-suited for incremental computations because it does not consider the locality of previously computed results.

To enable efficient re-use of previously run tasks, tasks should preferentially be scheduled on the same node where some or all of the data they use is stored. This is important, for instance, in case the Contraction phase needs to run using a combination of newly computed and memoized results, which happens when only a part of its inputs have changed. In addition to this requirement, the scheduler also has to provide some flexibility by allowing tasks to be scheduled on nodes that do not hold memoized results, otherwise it can lead to the presence of stragglers, i.e., individual poorly performing nodes that can drastically delay the overall job completion [20].

Based on these requirements, Incoop includes a new memoization-aware scheduler that strikes a balance between exploiting the locality of memoized results, and incorporating some flexibility to minimize the straggler effect. The main policy that the scheduler tries to implement is a location-aware policy that prevents the unnecessary movement of data, but at the same time it implements a simple work-stealing algorithm to adapt to varying resource availability. The scheduler works by maintaining a separate task queue for each node in the cluster (instead of a single task queue for all nodes), where each queue contains the tasks that should run in that node to maximally exploit the location of memoized results. Whenever a node requests more work, the scheduler dequeues the first task from the corresponding queue and assigns the task to the node for execution. In case the corresponding queue for the requesting node is empty, the scheduler tries to steal work from other task queues. The scheduling algorithm searches the task queues of other nodes, and steals a pending task from the task queue with maximum length. If there are multiple queues of maximum length, the scheduler steals the task that has the least amount memoized intermediate results. The scheduler thus takes the location of the memoized results into account, but falls back to a work-stealing approach to avoid stragglers and nodes running idle. Our experimental evaluation ($\S$7.6) shows the effectiveness of our approach.

6. ANALYSIS OF INCOOP

We analyze the asymptotic efficiency of Incoop. We consider two different runs: the initial-run of an Incoop computation, where we perform a computation with some input $I$, and a subsequent run or a dynamic update where we change the input from $I$ to $I'$ and perform the same computation with the new input. In the common case, we perform a single initial run followed by many dynamic updates. Using the information stored (e.g., inputs and outputs of tasks), each dynamic update identifies and executes the tasks that operate on the fresh (including changed) data while re-using the results of those that remain unaffected.

For the initial run, we define the overhead as the slowdown of Incoop compared to a conventional implementation of MapReduce such as with Hadoop. We show that the overhead depends on communications costs and if these are constant (i.e., independent of the input size), which they often are, it is also a constant. Our experiments (Section 7) show the overhead to be relatively small. We show that dynamic updates are dominated by the time it takes to execute fresh tasks that are affected by the changes to the input data, which for a certain class of computations and small changes is logarithmic in the size of the input.

In the analysis, we use the following terminology to refer to the three different types of computational tasks that form an Incoop computation: Map tasks, Contraction tasks (applications of the Combiner function), and Reduce tasks.
Our approach relies on computing hashes for (chunks of) data and communicating with a memoization server. We write $t_h$ for the time it takes to hash data and $t_m$ for the time it takes to send a simple, short message (exact contents to be specified later) to the memoization server. Our bounds depend on the total number of map tasks, written $N_M$ and the total number of reduce tasks written $N_R$. We write $n_i$ and $n_0$ to denote the total size of the input and output respectively, $n_m$ to denote the total number of key-value pairs emitted by the Map phase, and $n_{mk}$ to denote the set of distinct keys emitted by the Map phase.

For our time bounds, we will additionally assume that each Map, Combine, and Reduce function performs work that is asymptotically linear in the size of their inputs. We will additionally assume that the Combine function is monotonic, i.e., it produces and output that is no greater than its input. This assumption is satisfied in many applications, because Combiners often reduce the size of the data (e.g., a Combine function to compute the sum of values takes multiple values and outputs a single value).

Due to space restrictions, we present the complete proofs in Appendix B and here we only state our final theorems.

**Theorem 1 (Initial Run: Time and Overhead).** Assuming that Map, Combine, and Reduce functions take time asymptotically linear in their input size and that Combine functions are monotonic, the total time for performing an Incremental MapReduce computation in Incoop with an input size of $n_i$, where $n_{mk}$ key-value pairs are emitted by the Map phase is $O(t_{memo} \cdot (N_M + N_R + N_C)) = O(t_{memo} \cdot (n_i + n_m))$. This results in an overhead of $O(t_{memo}) = O(t_h + t_m)$ over conventional MapReduce.

**Theorem 2 (Initial Run: Space).** The total storage space for performing an Incoop computation with an input of size $n_i$, where $n_{mk}$ key-value pairs are emitted by the Map phase, and where Combine is monotonic is $O(n_i + n_m + n_0)$.

**Theorem 3 (Dynamic Update: Space and Time).** In Incoop, a dynamic update requires time

$$O \left( t_{memo} \cdot (N_M + N_C + N_R) + \sum_{a \in P} t(a) \right).$$

The total storage requirement is the same as an initial run.

**Theorem 4 (Number of Fresh Tasks).** If the Map function generates $k$ key-value pairs from a single input record, and the Combine function is monotonic, then the number of fresh tasks is at most $O(k \log n_m + k)$.

Taken together the last two theorems suggest that small changes to data will lead to execution of a small number of fresh tasks and based on the tradeoff between the memoization costs and the cost of executing a fresh tasks, speedups can be achieved in practice.

7. **IMPLEMENTATION AND EVALUATION**

We evaluate the effectiveness of Incoop for a variety of applications implemented in the traditional MapReduce programming model. In particular, we will answer the following questions:

- How does Incoop’s Inc-HDFS perform compared to HDFS? (§7.4)
- What performance benefits does Incoop provide for incremental workloads compared to the unmodified Hadoop implementation? (§7.5)
- How effective are the optimizations we propose in improving the overall performance of Incoop? (§7.6)
- What overheads does the memoization in Incoop impose when tasks are executed for the first time? (§7.7)

7.1 **Implementation**

We built our prototype of Incoop based on Hadoop-0.20.2. We implemented Inc-HDFS by extending HDFS with stable input partitioning, and incremental MapReduce by extending Hadoop with a finer granularity control mechanism and the memoization-aware scheduler.

The inc-HDFS file system provides the same semantics and interface for accessing all native HDFS calls. It employs a content-based chunking scheme which is computationally more expensive than the fixed-size chunking used by HDFS. As described in §3, the implementation minimizes the overhead using two optimizations: (i) we skip parts of the file contents when searching for chunk markers to reduce the number of fingerprint computations and enforce a minimum chunk size; and (ii) we parallelize the search for markers across multiple cores. To implement these optimizations, the data uploader client skips a fixed number of bytes after the last marker is found, and then spawns multiple threads that each compute the Rabin fingerprints over a sliding window on different parts of the content. For our experiments,
we set the number of bytes skipped to 40MB unless stated otherwise.

We implemented the memoization server using a wrapper around Memcached v1.4.5, which supports an in-memory key-value store. Memcached runs as a daemon process on the name node machine that acts as a directory server in Hadoop. Intermediate results memoized across runs are stored on inc-HDFS with the replication factor set to 1, and, in case of data loss, the intermediate results are recomputed. A major issue with any implementation of memoization is determining which intermediate results to remember and which intermediate results to purge. As in self-adjusting computation approaches, our approach is to cache the fresh results from the “last run”, i.e., those results that were generated or used by the last execution, and purge all the other obsolete results. This suffices to obtain the efficiency improvements proven in §7.7. We implement this strategy using a simple garbage collector that visits all cache entries and purges the obsolete results.

7.2 Applications and Data Generation

For the experimental evaluation, we use a set of applications in fields like machine learning, natural language processing, pattern recognition, and document analysis. Table 1 lists these applications. We chose these applications to demonstrate Incoop’s ability to efficiently execute both data-intensive (WordCount, Co-Matrix, BiCount), and computation-intensive (KNN and K-Means) jobs. In all cases, we did not make any changes to the original code.

All data-intensive applications take as input documents written in a natural language. In our benchmarks, we use a publicly available dataset with the contents of Wikipedia1. The computation-intensive applications take as input a set of points in a d-dimensional space. We generate this data synthetically by uniformly randomly selecting points from a 50-dimensional unit cube. To ensure reasonable running times, we chose all the input sizes such that the running time of each job would be around one hour in our cluster.

7.3 Measurements

Work and (parallel) time. For comparing different runs, we consider two types of measures, work and time, which are standard measures of comparing efficiency in parallel applications. Work refers to the total amount of computation performed by all tasks and measured as the total running of all tasks. (Parallel) Time refers to the amount of (end-to-end) time that it takes to complete a parallel computation. It is well-known that under certain assumptions a computation with W work can be executed on P processors in

<table>
<thead>
<tr>
<th>Application</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-Means Clustering (K-Means)</td>
<td>k-means clustering is a method of cluster analysis for partitioning n data points into k clusters, in which each observation belongs to the cluster with the nearest mean.</td>
</tr>
<tr>
<td>Word-Count</td>
<td>Word count determines the frequency of words in a document.</td>
</tr>
<tr>
<td>k-NN Classifier (KNN)</td>
<td>K-nearest neighbors classifies objects based on the closest training examples in a feature space.</td>
</tr>
<tr>
<td>Co-occurrence Matrix (CoMatrix)</td>
<td>Generates the Co-occurrence matrix, which is an N × N matrix where N is number of unique words in the corpus. A cell m_{ij} contains the number of times word w_i co-occurs with word w_j.</td>
</tr>
<tr>
<td>Bigram-Count (BiCount)</td>
<td>Bigram count measures the prevalence of a subsequence of two items within a given sequence.</td>
</tr>
</tbody>
</table>

Table 1: Applications used in the performance evaluation

To evaluate the overhead introduced by the content-based chunking in Inc-HDFS, we compare the throughput when

<table>
<thead>
<tr>
<th>Version</th>
<th>Skip Offset [MB]</th>
<th>Throughput [MB/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDFS</td>
<td>-</td>
<td>34.41</td>
</tr>
<tr>
<td>Incremental HDFS</td>
<td>20</td>
<td>32.67</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>34.19</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>32.04</td>
</tr>
</tbody>
</table>

Table 2: Throughput of HDFS and Inc-HDFS

uploading a dataset of 3 GB for HDFS and Inc-HDFS. In HDFS the chunk size is fixed at 64MB, while in Inc-HDFS, we vary the number of skipped bytes. The uploading client machine was co-located with the name node of the cluster, and we configured the parallel chunking code in Inc-HDFS to use 12 threads, i.e., one thread per core. The results of the experiments shown in Table 2 illustrate the effectiveness of our performance optimizations. Compared to plain HDFS, Inc-HDFS introduces only a minor throughput degradation due to the fingerprint computation that is required to perform content-based chunking.

For a smaller skip offset of 20MB, Inc-HDFS introduces a more noticeable overhead because Rabin fingerprints are computed for a larger fraction of the data, which results in a degradation of overall throughput. On the other hand, for a larger skip offset of 60MB, we again see a small performance degradation despite the smaller computational overhead for fingerprinting. This is due to the fact that a larger skip offset increases the average chunk size, resulting in a lower total number of chunks for an input file. As a consequence, less work can be done in parallel towards the end of the upload. For a skip offset of 40MB, however, the Inc-HDFS throughput is similar to HDFS because it strikes a balance between the computational overhead of fingerprint computations and opportunities for parallel processing of data blocks during the upload to the distributed file system.

7.5 Run-Time Speedups

![Figure 5: Work speedups versus change size.](image)

![Figure 6: Time speedups versus change size.](image)

Figures 5 and 6 show the work and time speedups measure as the work and time of the dynamic run with Incoop divided by those with Hadoop. From these experimental results we can observe the following: (i) Incoop achieves substantial performance gains for all applications when there are incremental changes to the input data. In particular, work and time speedups vary between over a fold of 1000 and 3, for incremental modifications ranging from 0% to 25% of data. (ii) We observe higher speedups for computation-intensive applications (K-Means, KNN) than for data-intensive applications (WordCount, Co-Matrix, BiCount). This is consistent with the approach (and the analysis in Appendix B) because the approach benefits computation-intensive tasks by avoiding unnecessary computations by re-using results. (iii) Both work and time speedups decrease as the size of the incremental change increases because larger changes allow fewer computation results from previous runs to be re-used. With very small changes, however, speedups in total work are not realized fully as speedups in parallel time; this is expected because decreasing the total amount of work dramatically (e.g., by a factor 1000) reduces amount of parallelism, causing the scheduling overheads to be more pronounced. Indeed, as the size of the incremental change increases, the gap between the work speedup and time speedup closes quickly.

The aforementioned examples all consider inputs of a fixed size. We also ran experiments that varied the size inputs, which show that similar results hold. In particular, Figure 7 illustrates the time to run Incoop and Hadoop under a single chunk change for the Co-Matrix application as we double the input size from our starting input size. This shows that the relative improvements hold for various different input sizes.

7.6 Effectiveness of Optimizations

We evaluate the effectiveness of the optimizations in improving the overall performance of Incoop by considering (i) the granularity control with the introduction of the Contraction phase; and (ii) the scheduler modifications to minimize unnecessary data movement.

**Granularity control.** To evaluate the effectiveness of the Contraction phase, we consider the two different levels of memoization in Incoop: (i) the coarse grained, task-level memoization performed in the implementation denoted as Task, and (ii) the fine-grained approach that adds the Contraction phase in the implementation denoted as Contrac-
tion. Figure 8 shows our time measurements with CoMatrix as a data-intensive application and KNN as a computation-intensive application. The effectiveness of Contraction is negligible with KNN while significant in CoMatrix. The reason for negligible improvements with KNN is that in this application, reducers perform relatively inexpensive tasks and thus benefits little from the Contraction phase. The results illustrate that even when not helpful, the Contraction phase does not degrade efficiency and can significantly improve performance for other applications.

Figure 8: Performance gains comparison between Contraction and task variants

7.7 Overheads

The memoization performed in Incoop introduces runtime overheads for the first run of tasks when no results from previous runs can be reused. Also, memoizing intermediate task results imposes an additional space usage. We measured both types, performance and space overhead, for each application and present the results in Figure 10.

Performance overhead. We measure the worst-case performance overhead by capturing the run-time for the first job run with respect to Hadoop. Figure 10(a) depicts the performance penalty for both the Task and the Contraction memoization based approach. The overhead varies from 5% – 22%, and, as expected, it is lower for computation intensive applications such as K-Means, KNN, since their run-time is dominated by the actual processing time rather than storing, retrieving and transferring data. For the data intensive applications such as WordCount, Co-Matrix, Bi-Count, the first run with Task level memoization is faster than Contraction memoization. This difference in performance can be attributed to the extra processing overheads for all levels of the tree formed in the Contraction phase. Importantly, this performance overhead is a one-time cost and the subsequent runs benefit with high speedup.

Space overhead. We measure the space overhead relative to input size by quantifying the space used for remembering the intermediate computation results. Figure 10(b) illustrates the space overhead as a factor of the input data size with Task- and Contraction-level memoization. The results show that the Contraction-level memoization requires more space, which was expected because it stores results for all levels of the Contraction tree. Overall, space overhead varies substantially depending on the application, and can be as high as a 9X factor (CoMatrix application). However, our approach for garbage collection (Section 7.1) prevents the storage utilization from growing over time.

8. RELATED WORK

Our work builds on contributions from several different fields, which we briefly survey.

Dynamic algorithms. In the algorithms community, researchers designed dynamic algorithms that permit modifications or dynamic changes to their input, and efficiently
update their output when such changes occur. Several surveys illustrate the vast literature on dynamic algorithms [6]. This research shows that dynamic algorithms can be asymptotically, often by a near-linear factor, more efficient than their conventional counterparts. In large-scale systems, this asymptotic difference can yield significant speedups. Dynamic algorithms can, however, be difficult to develop and implement even for simple problems; some problems took years of research to solve and many remain open.

**Programming language-based approaches.** In the programming languages community, researchers developed incremental computation techniques to achieve automatic incrementalization. The basic idea is to automatically translate a program that is conventionally defined as mapping between an input and an output to a program that permits modifications to its input while updating its output. Incremental computation attracted significant attention and many techniques have been proposed (e.g. [19]). Being automatic and hiding the mechanism for incrementalization, this approach can dramatically simplify software development. Building on this work, more recent proposals for self-adjusting computation [1] are based on general-purpose techniques for expressing incremental computation at a high-level and deriving efficient executables by using compilers specifically developed for this purpose. However, existing approaches to self-adjusting computation consider sequential, non-distributed, and uniprocessor computation models (e.g. [10]) and therefore are not applicable to the distributed execution model of the MapReduce framework.

**Incremental database view maintenance.** There is substantial work from the database community on incrementally updating a database view (i.e., a predetermined query on the database) as the database contents evolve. The techniques used by these systems can either directly operate on the database internals to perform these incremental updates, or rely on SQL queries that efficiently compute the modifications to the database view, and that are issued upon the execution of a database trigger [5]. Even though Incoop shares the same goals and principles with incremental view maintenance, it differs substantially in the techniques that are employed, since the latter exploits database-specific mechanisms and semantics.

**Large-scale incremental parallel data processing.** There are several systems for performing incremental parallel computations with large data sets. We broadly divide them into two categories: non-transparent and transparent approaches. Examples of non-transparent systems include Google’s Percolator [17], which is now used to maintain their web index. In Percolator, the programmer writes a program in an event-driven programming model, where an application is structured as a series of observers. Observers are triggered by the system whenever user-specified data changes. Similarly, Yahoo’s continuous bulk processing (CBP) [12] proposes a new data-parallel programming model, inspired by MapReduce, which introduces new primitives to store and reuse prior state for incremental processing. In particular, loop-back flows redirect the output of a stage as the input for subsequent runs. Despite the performance benefits these systems brought, there are two drawbacks with these approaches that are addressed by our proposal. The first is that they depart from the MapReduce programming paradigm and therefore require changes to the large existing base of MapReduce programs. The second, more fundamental problem is that they require the programmer to devise a dynamic algorithm in order to efficiently process data in an incremental manner.

Examples of transparent approaches include DryadInc [18], which extends Dryad to automatically identify redundant computations by caching previously executed tasks. One limitation of this basic approach is that it can only reuse common identical sub-DAGs of the original computation, which can be insufficient to achieve efficient updates. To improve efficiency the paper suggests the programmers specify additional merge functions. Another similar system called Nectar [9] caches prior results at the coarser granularity of entire LINQ sub-expressions. The technique used to achieve this is to automatically rewrite LINQ programs to facilitate caching. Finally, although not fully transparent, Haloop [4] provides task-level memoization techniques for memoization in the context of iterative data processing applications. The major difference between the aforementioned transparent approaches and our proposal is that we use a well-understood set of principles from related work to eliminate the cases where task-level memoization provides poor efficiency. To this end, we provide techniques for increasing the effectiveness of task-level memoization via stable input partitions.

![Figure 10: Overheads imposed by Incoop in comparison to Hadoop](image-url)
and by using a more fine-grained memoization strategy than the granularity of Map and Reduce tasks.

Our own short position paper [3] makes the case for applying techniques inspired by self-adjusting computation to large-scale data processing in general, and uses MapReduce as an example. This position paper, however, models MapReduce in a sequential, single-machine implementation of self-adjusting computation called CEAL [10], and does not offer anything close to a full scale distributed design and implementation such as we describe here.

**Stream processing systems.** Comet [11] introduces the Batched Stream Processing (BSP) model, where input data is modeled as a stream, with queries being triggered upon bulk appends to the stream. In particular, the interface provided by Comet enables exploiting temporal and spatial correlations in recurring computations by defining the notion of a query series. Within a query series, the execution will automatically leverage the intermediate results of previous invocations of the same query on an overlapping window of the data, thereby exploiting temporal correlations. Further, by aligning multiple query series to execute together when new bulk updates occur, Comet exploits spatial correlations by removing redundant I/O or computation across those queries. In contrast to Comet, we are compatible with the MapReduce model and focus on several aspects like controlling task granularity or input partitioning that do not arise in Comet’s model.

NOVA [15] is a workflow manager recently proposed by Yahoo!, designed for the incremental execution of Pig programs upon continually-arriving data. NOVA introduces a new layer called the workflow manager on the top of the Pig/Hadoop framework. Much like the work on incremental view maintenance, the workflow manager rewrites the computation in such a way that identifies the parts of the computation affected by incremental changes and produces the necessary update function that runs on top of the existing Pig/Hadoop framework. However, as noted by the authors of NOVA, an alternative, more efficient design would be to modify the underlying Hadoop system to support this functionality. In our work, and particularly with our case study of incremental processing of Pig queries, we explore precisely this alternative design of adding lower-level support for reusing previous results. Furthermore, our work is broader in that it transparently benefits all MapReduce computations, and not only continuous Pig queries.

9. CONCLUSION

In this paper, we presented Incoop, a novel MapReduce framework for large-scale incremental computations. Incoop proposes several novel techniques to maximize the re-use of results from a previous computation. In particular, Incoop incorporates content-based chunking to the file system to detect incremental changes in the input file and to partition the data so as to maximize re-use, it adds a contraction phase to control the granularity of tasks in the reduce phase, and a new scheduler that takes the location of previously computed results into account. We implemented Incoop as an extension to Hadoop. Our performance evaluation shows that Incoop can improve efficiency in incremental runs (common case), at a modest cost in the initial, first run (uncommon case) where no computations can be reused.

10. REFERENCES

APPENDIX

A. CASE STUDIES

The success of MapReduce paradigm enables our approach to transparently benefit an enormous variety of bulk data processing workflows. In particular, and aside from the large number of existing MapReduce programs, MapReduce is also being used as an execution engine for other systems. In this case, Incoop will also transparently benefit programs written for these systems.

In this section, we showcase two workflows where we use Incoop to transparently benefit applications and systems whose importance has justified the development of specific solutions for efficient incremental processing in their context, namely incremental log processing and incremental query processing.

A.1 Incremental Log Processing

Log processing is an essential workflow in Internet companies, where various logs are often analyzed in multiple ways on a daily basis [13]. For example, in the area of click log analysis, traces collected from various web server logs are aggregated in a single repository and then processed for various purposes, from simple statistics like counting clicks per user, or more complex analyzes like click sessionization.

To perform incremental log processing, we integrated Incoop with Apache Flume ² – a distributed and reliable service for efficiently collecting, aggregating, and moving large amounts of log data. In our setup, Flume uploads the data and dumps it into the Inc-HDFS repository. Then, Incoop performs the analytic processing incrementally, leveraging previously computed intermediate results.

We evaluate the performance of using Flume in conjunction with Incoop for incremental log processing by comparing its runtime with the corresponding runtime when using Hadoop. For this experiment, we perform document analysis on an initial set of logs, and then append new log entries to the input, after which we process the resulting larger collection of logs incrementally. In Figure 11, we depict the speedup for running Incoop as a function of the size of the new logs that are appended after the first run. Incoop achieves a speedup of a factor of 4 to 2.5 with respect to Hadoop when processing incremental log append of size of 5% to 25% of the initial input size, respectively.

A.2 Incremental Query Processing

We showcase incremental query processing as another workflow that exemplifies the potential benefits of Incoop. Incremental query processing is an important workflow in Internet companies, where the same query is processed frequently for an incrementally changing input data set [15]. We integrated Incoop with Pig to evaluate the feasibility of incremental query processing. Pig [16] is a platform to analyze large data sets built upon Hadoop. Pig provides Pig Latin, an easy-to-use high-level query language similar to SQL. The ease of programming and scalability of Pig made the system very popular for very large data analysis tasks, which are conducted by major Internet companies today.

Since Pig programs are compiled down to multi-staged MapReduce jobs, the integration of Incoop with Pig was seamless, just by using Incoop as the underlying execution engine for incrementally executing the multi-staged MapReduce jobs. We evaluate two Pig applications, word count and the PigMix ³ scalability benchmark, to measure the effectiveness of Incoop. We observe a runtime overhead of around 15% for first run, and a speedup of a factor of around 3 for an incremental run with unmodified input. The detailed result breakdown is shown in Table 3.

![Figure 11: Speedup for incremental log processing](image)

Table 3: Results for incremental query processing

<table>
<thead>
<tr>
<th>Application</th>
<th>Features</th>
<th>M/R stages</th>
<th>Overhead</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Count</td>
<td>Group by, Order by, Filter</td>
<td>3</td>
<td>15.65 %</td>
<td>2.84</td>
</tr>
<tr>
<td>PigMix scalability benchmark</td>
<td>Group by, Filter</td>
<td>1</td>
<td>14.5 %</td>
<td>3.33</td>
</tr>
</tbody>
</table>

B. ANALYSIS OF INCOOP (PROOFS)

**Theorem 5 (Initial Run: Time and Overhead).**

Assuming that Map, Combine, and Reduce functions take time asymptotically linear in their input size and that Combine functions are monotonic, total time for performing an Incremental MapReduce computation in Incoop with an input of size $n_i$, where $n_{key-value}$-pair is emitted by the Map phase is $O(t_{memo} \cdot (N_M + N_R + N_C)) = O(t_{memo} \cdot (n_i + n_m))$. This results in an overhead of $O(t_{memo}) = O(t_h + t_m)$ over conventional MapReduce.

**Proof.** Memoizing a task requires 1) computing the hash of each input, and 2) sending a message to the memoization server containing the triple consisting of the task id, the input hash and the location of the computed result. Given the time for hashing an input chunk $t_h$ and the time for sending a message $t_m$, this requires $t_{memo} = t_h + t_m \in O(t_h + t_m)$ for each task of the job. Memoization therefore causes $O(t_h + t_m)$ per task slowdown. To compute the total slowdown we bound the number of tasks.

The number of Map and Reduce tasks in a particular job can be derived from the input size and the number of distinct

²Apache Flume: https://github.com/cloudera/flume
³Apache PigMix: http://wiki.apache.org/pig/PigMix
keys that are emitted by the Map function: the Map function is applied to splits that consist of one or more input chunks, and each application of the Map function is performed by one Map task. Hence, the number of Map tasks \( N_M \) is in the order of input size \( O(n_i) \). In the Reduce phase, each Reduce task processes all previously emitted key-value pairs for at least one key, which results in at most \( N_R = n_{mk} \) reduce tasks. To bound the number of contraction tasks, we note that the contraction phase builds a tree whose leaves are the output data chunks of the Map phase, whose internal nodes each has at least two children. Since there are at most \( n_{mk} \) pairs output by the Map phase, the total number of reduce tasks is bounded by \( n_{mk} \). Hence the total number of contraction tasks \( N_C \) is \( O(n_{mk}) \). Since the number of reduce tasks is bounded by \( n_{mk} \leq n_m \), the total number of tasks is \( O(n_i + n_m) \).

**Theorem 6 (Initial Run: Space).** Total storage space for performing an Incoop computation with an input of size \( n_i \), where \( n_{mk} \) key-value pairs are emitted by the Map phase, and where Combine is monotonic is \( O(n_i + n_m + n_O) \).

**Proof.** In addition to the input and the output, Incremental MapReduce requires additionally storing the output of the map, contraction, and reduce tasks. Since Incoop only keeps data from the most recent run (initial or dynamic run), we only use storage for remembering the task output from the most recent run. The output size of the map tasks is bounded by \( n_{mk} \). With monotonic Combine functions, the size of the output of Combine tasks is bounded by \( O(n_{mk}) \). Finally, the storage needed for reduce tasks is the bounded by the size of the output.

**Theorem 7 (Dynamic Update: Space and Time).** In Incoop, a dynamic update requires time

\[
O\left(t_{\text{memo}}(N_M + N_C + N_R) + \sum_{a \in F} t(a)\right).
\]

The total storage requirement is the same as an initial run.

**Proof.** Consider Incoop performing an initial run with input \( I \) and changing the input to \( I' \) and then performing a subsequent run (dynamic update). During the dynamic update, tasks with the same type and input data will re-use the memoized result of the previous, avoiding recomputation. Thus, only the fresh tasks need to be executed, which takes

\[
O\left(\sum_{a \in F} t(a)\right), \quad \text{where } F \text{ is the set of changed or new (fresh) Map, Contract and Reduce tasks, respectively, and } t(\cdot) \text{ denotes the processing time for a given task.}
\]

Re-using tasks will, however, require an additional check with the memo server we will pay a cost of \( t_{\text{memo}} \) for all re-used tasks.

In the common case, we expect that execution of fresh tasks to dominate the time for dynamic updates, because \( t_{\text{memo}} \) is a relatively small constant. The time for dynamic update is therefore likely to be determined by the number of fresh tasks that are created as a result of a dynamic change. It is in general difficult to bound the number of fresh tasks, because it depends on the specifics of the application. As a trivial example, consider, inserting a single key-value pair to the input. In principle, the new pair can force the Map function to generate a very large number of new key-value pairs, which can then require performing many new reduce tasks. In many cases, however, small changes to the input lead only small changes in the output of the Map, Combine, and Reduce functions, e.g., the Map function can use one key-value pair to generate several new pairs, and the Combine function will typically combine these, resulting in a relatively small number of fresh tasks. As a specific case, assume that the Map function generates \( k \) key-value pairs from a single input record, and that the Combine function monotonically reduces the number of key-value pairs.

**Theorem 8 (Number of Fresh Tasks).** If the Map function generates \( k \) key-value pairs from a single input record, and the Combine function is monotonic, then the number of fresh tasks is at most \( O(k \log n_m + k) \).

**Proof.** At most \( k \) contraction tasks at each level of the contraction tree will be fresh, and \( k \) fresh reduce tasks will be needed. Since the depth of the contraction tree is \( n_m \), the total number of fresh tasks will therefore be \( O(k \log n_m + k) = O(k \log n_m) \).