Random Variables

Family		Range	Density	Mean	Variance	Generating function
Poisson	$\mathcal{P}(\lambda)$	$0,1,\ldots$	$\mathbb{P}(X=r)=\dfrac{\lambda^r e^{-\lambda}}{r!}$	λ	λ	$\mathbb{E}(s^X)=e^{\lambda(s-1)}$
Geometric	$\mathrm{Geom}(p)$	$1,2,\ldots$	$\mathbb{P}(X=r)=p(1-p)^{r-1}$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$	$\mathbb{E}(s^X)=\dfrac{sp}{1-s(1-p)}$
Binomial	$\mathrm{Bin}(n,p)$	$0,1,\ldots n$	$\mathbb{P}(X=r)=\binom{n}{r}p^r(1-p)^{n-r}$	np	$np(1-p)$	$\mathbb{E}(s^X)=\big(ps+(1-p)\big)^n$
Uniform	$\mathrm{U}[1,n]$	$1,2,\ldots n$	$\mathbb{P}(X=r)=\dfrac{1}{n}$	$\dfrac{n+1}{2}$	$\dfrac{n^2-1}{12}$	$\mathbb{E}(s^X)=\dfrac{s(1-s^n)}{n(1-s)}$
Negative Binomial	$\mathrm{NegBin}(\lambda,\nu)$	$0,1,\ldots$	$\mathbb{P}(X=r)=\binom{r+\lambda/\nu-1}{r}e^{-\lambda}(1-e^{-\nu})^r$	$\dfrac{\lambda}{\nu}(e^\nu-1)$	$\dfrac{\lambda}{\nu}(e^\nu-1)e^\nu$	$\mathbb{E}(s^X)=\left(\dfrac{e^{-\nu}}{1-s(1-e^{-\nu})}\right)^{\lambda/\nu}$
Uniform	$\mathrm{U}[a,b]$	$[a,b]$	$f(x)=\dfrac{1}{b-a}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	$\mathbb{E}(e^{tX})=\dfrac{e^{bt}-e^{at}}{(b-a)t}$
Exponential	$\mathrm{Exp}(\lambda)$	$\mathbb{R}^+$	$f(x)=\lambda e^{-\lambda x}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$	$\mathbb{E}(e^{tX})=\dfrac{\lambda}{\lambda-t}$
Gamma	$\Gamma(\gamma,\lambda)$	$\mathbb{R}^+$	$f(x)=\dfrac{\lambda^\gamma x^{\gamma-1}e^{-\lambda x}}{\Gamma(\gamma)}$	$\dfrac{\gamma}{\lambda}$	$\dfrac{\gamma}{\lambda^2}$	$\mathbb{E}(e^{tX})=(1-t/\lambda)^{-\gamma}$
Normal	$\mathrm{N}(\mu,\sigma^2)$	$\mathbb{R}$	$f(x)=\dfrac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	μ	σ^2	$\mathbb{E}(e^{tX})=\exp(t\mu+\frac{1}{2}t^2\sigma^2)$
MVN	$\mathrm{N}(\mu,\Sigma)$	$\mathbb{R}^n$	$f(x)=\dfrac{\exp\big[-\frac{1}{2}(x-\mu)^\mathsf{T}\Sigma^{-1}(x-\mu)\big]}{\sqrt{(2\pi)^n\det\Sigma}}$	μ	Σ	$\mathbb{E}(e^{t^\mathsf{T}X})=\exp(t^\mathsf{T}\mu+\frac{1}{2}t^\mathsf{T}\Sigma t)$
Cauchy	Cauchy	$\mathbb{R}$	$f(x)=\dfrac{1}{\pi}\dfrac{1}{1+x^2}$	undefined	undefined	$\mathbb{E}(e^{tX})=\infty$ for $t\neq 0$

If N_t is a Poisson process of rate λ then the time until $N_t=1$ is $\mathrm{Exp}(\lambda)$, the time until $N_t=n$ is $\Gamma(n,\lambda)$, and N_t is $\mathrm{Poisson}(\lambda t)$.

If N_t is a Markov birth process with birth rates $\lambda+n\mu$ then N_t is $\mathrm{NegBin}(\lambda t,\nu)$. If $\lambda=\nu$ then N_t+1 is $\mathrm{Geom}(e^{-\nu})$.

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More Random Variables

Family		Range	Distribution	Notes
Chi-squared	χ^2_n	$\mathbb{R}^+$	$\chi^2_n \sim \sum^n \mathrm{N}(0,1)^2$	$\chi^2_n \sim \Gamma(\frac{n}{2}, \frac{1}{2})$
Student t	t_n	$\mathbb{R}$	$t_n \sim \dfrac{N(0,1)}{\sqrt{\chi^2_n/n}}$	$t_1 \sim$ Cauchy. $\mathbb{E}t_n = 0$ if $n > 1$; $\operatorname{Var} t_n = \dfrac{n}{n-2}$ if $n > 2$.
F ratio	$F_{m,n}$	$\mathbb{R}^+$	$F_{m,n} \sim \dfrac{\chi^2_m/m}{\chi^2_n/n}$	$\mathbb{E}F_{m,n} = \dfrac{m}{m-2}$ for $m > 2$.
Weibull	Weibull(α)	$\mathbb{R}^+$	$f(x) = \alpha x^{\alpha-1} e^{-x^\alpha}$	Weibull(α) $\sim \mathrm{Exp}(1)^{1/\alpha}$
Beta	Beta(r,s)	$[0,1]$	$f(x) = \dfrac{x^{r-1}(1-x)^{s-1}}{\beta(r,s)}$	Beta(r,s) $\sim \left(1 + \dfrac{\Gamma(s,\lambda)}{\Gamma(r,\lambda)}\right)^{-1}$. Mean $\dfrac{r}{r+s}$.
Logistic	Logistic	$\mathbb{R}$	$f(x) = \dfrac{e^x}{(1+e^x)^2}$	Logistic $\sim \log\big(\mathrm{Pareto}(1) - 1\big)$. $\mathbb{E}(e^{tX}) = \beta(1+t, 1-t)$.
Pareto	Pareto(α)	$[1,\infty)$	$f(x) = \alpha x^{-(\alpha+1)}$	Pareto(α) $\sim e^{\mathrm{Exp}(\alpha)} \sim \mathrm{Beta}(\alpha,1)^{-1}$. $\mathbb{E}X = \infty$ if $\alpha \le 1$; $\operatorname{Var} X = \infty$ if $\alpha \le 2$.

$$n! \sim \sqrt{2\pi} n^{n+1/2} e^{-n}. \quad \Gamma(n) = (n-1)!. \quad \beta(r,s) = \frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)}.$$

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