Network Performance, GZ02 and M033, 2009/2010
Answer ALL questions from part A and ANY TWO questions from part B
Marks for each part of each question are indicated in square brackets
Calculators are permitted

You may find useful the following standard formulae. If $0 \le \rho < 1$, then
$1+\rho+\rho^2+\cdots = 1/(1-\rho)$, and $\rho+2\rho^2+3\rho^3+\cdots = \rho/(1-\rho)^2$.

Part A

1. I have taken a set of measurements of n predictor variables $w_1,\ldots,w_n$ and n response variables $X_1,\ldots,X_n$. I believe that the response variables are independent, and that $X_i \sim \text{Exp}(\lambda w_i)$, where λ is unknown. Calculate the maximum likelihood estimate for λ.

[11 marks]

2. What is meant by an $M/M/1/B$ queue? Supposing a $M/M/1/B$ queue is currently empty, let t be the expected time until it becomes full. Write down a set of equations that one could solve to find t.

[11 marks]

3. Find the average number of active calls on an Erlang link with C circuits, arrival rate λ and mean call duration m.

[12 marks]

Part B

4. This question concerns a simple slotted-time model for retransmissions by a node in a wireless network.

   The node keeps track of the number of transmission failures experienced by the packet it is currently trying to send. In each timeslot, the node may stay silent or it may attempt transmission; if it attempts transmission, it may either succeed or fail. If it succeeds, then it starts over with a new packet in the next timeslot. If it fails, it will retransmit the current packet in a future timeslot.

   Mathematically, let X_n be the number of transmission failures experienced by the current packet up to the beginning of timeslot n; then $X_{n+1} = X_n$ if the node stays silent, $X_{n+1} = 0$ if its transmit attempt succeeds, $X_{n+1} = X_n + 1$ if its transmit attempt fails. The probability of attempting transmission in timeslot n is $1/2^{X_n}$. The probability that a transmit attempt fails is q, and successes/failures are assumed to be independent of X_n and of what has happened in previous timeslots.

   (a) Find the distribution of the backoff, i.e. the wait until the next transmission attempt, after a packet has just experienced its kth failure. Show that the average backoff doubles after each transmission failure.

   (b) Draw the state space diagram for the Markov chain X_n. For what values of q is it stable? Calculate the equilibrium distribution. Show that, in equilibrium, the probability of attempting transmission is $p = (1-2q)/(1-q)$.

[33 marks]

5. (a) In a model for Dynamic Alternative Routing of calls on a fully-connected network of links each with C circuits, the equation

   $$B = E\big(\lambda/\mu + 2\lambda B(1-B)/\mu, C\big)$$

   can be used to compute the probability B that a given link has all its circuits busy. Explain the terms in the equation, and describe how to solve it numerically.

   (b) Consider the following simple slotted-time model for a wireless network consisting of N nodes. In each timeslot, the probability that a node attempts transmission is p. The transmission succeeds if none of the other nodes is also attempting transmission, otherwise it fails. Let the probability of failure be q. A careful analysis of the retransmission mechanism (in Question 4) shows that $p = (1-2q)/(1-q)$.

   Write down an expression for q in terms of p, and use this to find a fixed-point equation for q. Compute the numerical solution for q when $N = 10$. Also compute the probability that there is a successful transmission in a given timeslot (i.e. the total throughput of the wireless network).

[33 marks]

6. In the standard processor-sharing model of TCP, we assume that when a link with speed C is used by n TCP flows then each flow gets throughput $\theta(n) = C/n$. However, cable network engineers have found that, because of inefficiencies in the MAC protocol for cable access links, the achievable utilization is lower when several flows are sharing the link: each flow gets throughput $\theta(1) = C$ when there is one flow, and $\theta(n) = Cu/n$ when there are $n \geq 2$ flows, for some constant $0 < u < 1$.

   (a) Calculate the average flow completion time, as a function of the arrival rate λ, the mean job size m, the link speed C and the parameter u. You may find the formulae on the front page useful.

   Cable network engineers have proposed that it would be better to serve flows with the first-come-first-served (FCFS) discipline, i.e. the job at the head of the queue gets throughput C and the other jobs have to wait their turn. This way, the system does not suffer the problem of sharing inefficiency.

   (b) Calculate the average flow completion time under FCFS discipline.

   (c) It can be shown that the average flow completion time is smaller under FCFS. Explain briefly how FCFS might nevertheless lead to *worse* quality of service for some users.

[33 marks]

7. The MAC protocol for a cable access uplink works as follows. Time is slotted, and each timeslot is divided into a request subslot, a reply subslot, and a data subslot. In the request subslot, users with packets to send can signal a request to the cable head-end (CHE). The CHE chooses one of these users, and in the reply subslot the CHE signals its choice. In the data subslot, the chosen user transmits his packet. However, request signals can interfere with each other, and when this happens the timeslot is wasted and users have to wait until the next timeslot to re-signal their requests. To avoid this problem, the MAC protocol has an additional request mechanism: the chosen user in timeslot n can append a request for timeslot $n+1$ to his packet transmission in timeslot n, and the CHE will grant it. This has the effect of giving priority to ongoing flows. Of course, it only works if the chosen user has another packet ready to send.

   We may model this as follows. Let x be the throughput of a user, in packets per second. There is some minimum throughput x_{min} such that if $x > x_{\text{min}}$ then the priority mechanism works and the user experiences round trip time RTT_0. If $x < x_{\text{min}}$ then there is not enough data for the priority mechanism to work, so the user experiences round trip time $\text{RTT}_0 + \Delta$, for some constant $\Delta > 0$.

   Write down a drift model for the throughput of a TCP flow using this cable access link and subject to a fixed packet drop probability p. Sketch the drift diagram. Find any fixed points. Describe the behaviour that the drift model predicts.

[33 marks]

END OF PAPER