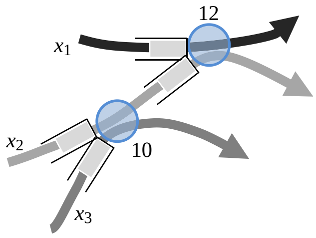
Example sheet 6

Resource allocation

Network Performance—DJW—2011/2012

Question 1. Here is a picture of two servers, and the paths taken by three classes of request. Requests of type i arrive at rate x_i per second. One server serves 10 requests per second, the other serves 12 requests per second. Completed requests of class 1, 2 and 3 earn £6, £5 and £4 respectively. Let the offered arrival rates be $(x_1, x_2, x_3) = (6, 9, 3)$ requests per second.



(i) Write down equations for the feasible region of this system. Which point in the feasible region achieves maximum revenue?

(ii) Given the specified arrival rates, the servers are overloaded, and so some requests must be dropped. What fraction of jobs of each class should be dropped, in order to maximize revenue? What revenue is achieved?

(iii) For the sake of simplicity, the operator decides that each server should run first-come–first-served. This means, for example, that if $x_1 + x_2 > 10$ then it drops a fraction $(x_1 + x_2 - 10)/(x_1 + x_2)$ of incoming requests, the same fraction for both request classes, and so the rate at which jobs of class i are admitted is

$$y_i = \frac{x_1 + x_2 - 10}{x_1 + x_2} x_i.$$

What revenue is achieved by this rule? *Note that first-come–first-served may end up wasting service effort, by serving jobs of class 2 at the upstream server only to drop them at the downstream server. A well-designed admission control policty would not do this.*

(iv) Suppose that drop rates are chosen so as to maximize the weighted α-fair utility function

$$6\frac{y_1^{1-\alpha}}{1-\alpha} + 5\frac{y_2^{1-\alpha}}{1-\alpha} + 4\frac{y_3^{1-\alpha}}{1-\alpha}$$

where y_i is the completion rate for jobs of class i. What revenue is achieved by this rule at $\alpha = 0$? $\alpha = 1$? $\alpha = 2$? *Note that for $\alpha = 1$ one should replace $y^{1-\alpha}/(1-\alpha)$ by $\log y$.*

Question 2. Explain what is meant by "TCP is a distributed mechanism for finding a utility-maximizing rate allocation of capacity in the Internet."

TCP corresponds to a utility function with $\alpha = 2$ and with weight $2/\mathsf{RTT}^2$. Suppose we wanted $\alpha = 1$ instead (which corresponds to replacing $y^{1-\alpha}/(1-\alpha)$ in the utility function by $\log y$). Find a window-based congestion control scheme which achieves this.