

# Random Variables

Family	Range	Density	Mean	Variance	Generating function	
Poisson	$\mathcal{P}(\lambda)$	$0, 1, \dots$	$\mathbb{P}(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}$	$\lambda$	$\lambda$	$\mathbb{E}(s^X) = e^{\lambda(s-1)}$
Geometric	$\text{Geom}(p)$	$1, 2, \dots$	$\mathbb{P}(X = r) = p(1-p)^{r-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\mathbb{E}(s^X) = \frac{sp}{1-s(1-p)}$
Binomial	$\text{Bin}(n, p)$	$0, 1, \dots, n$	$\mathbb{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$	$np$	$np(1-p)$	$\mathbb{E}(s^X) = (ps + (1-p))^n$
Uniform	$U[1, n]$	$1, 2, \dots, n$	$\mathbb{P}(X = r) = \frac{1}{n}$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$	$\mathbb{E}(s^X) = \frac{s(1-s^n)}{n(1-s)}$
Negative Binomial	$\text{NegBin}(\lambda, \nu)$	$0, 1, \dots$	$\mathbb{P}(X = r) = \binom{r + \lambda/\nu - 1}{r} e^{-\lambda} (1 - e^{-\nu})^r$	$\frac{\lambda}{\nu}(e^\nu - 1)$	$\frac{\lambda}{\nu}(e^\nu - 1)e^\nu$	$\mathbb{E}(s^X) = \left( \frac{e^{-\nu}}{1 - s(1 - e^{-\nu})} \right)^{\lambda/\nu}$
Uniform	$U[a, b]$	$[a, b]$	$f(x) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\mathbb{E}(e^{tX}) = \frac{e^{bt} - e^{at}}{(b-a)t}$
Exponential	$\text{Exp}(\lambda)$	$\mathbb{R}^+$	$f(x) = \lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\mathbb{E}(e^{tX}) = \frac{\lambda}{\lambda - t}$
Gamma	$\Gamma(\gamma, \lambda)$	$\mathbb{R}^+$	$f(x) = \frac{\lambda^\gamma x^{\gamma-1} e^{-\lambda x}}{\Gamma(\gamma)}$	$\frac{\gamma}{\lambda}$	$\frac{\gamma}{\lambda^2}$	$\mathbb{E}(e^{tX}) = (1 - t/\lambda)^{-\gamma}$
Normal	$N(\mu, \sigma^2)$	$\mathbb{R}$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$	$\mu$	$\sigma^2$	$\mathbb{E}(e^{tX}) = \exp(t\mu + \frac{1}{2}t^2\sigma^2)$
MVN	$N(\mu, \Sigma)$	$\mathbb{R}^n$	$f(x) = \frac{\exp[-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)]}{\sqrt{(2\pi)^n \det \Sigma}}$	$\mu$	$\Sigma$	$\mathbb{E}(e^{t^\top X}) = \exp(t^\top \mu + \frac{1}{2}t^\top \Sigma t)$
Pareto	$\text{Pareto}(\alpha)$	$[1, \infty)$	$f(x) = \alpha x^{-(\alpha+1)}$	$\alpha(\alpha-1)^{-1}$ , if $\alpha > 1$	$\alpha(\alpha-1)^{-2}(\alpha-2)^{-1}$ , if $\alpha > 2$	$\mathbb{E}(e^{tX}) = \infty$ for $t > 0$
Cauchy	Cauchy	$\mathbb{R}$	$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$	undefined	undefined	$\mathbb{E}(e^{tX}) = \infty$ for $t \neq 0$

If  $N_t$  is a Poisson process of rate  $\lambda$  then the time until  $N_t = 1$  is  $\text{Exp}(\lambda)$ , the time until  $N_t = n$  is  $\Gamma(n, \lambda)$ , and  $N_t$  is  $\text{Poisson}(\lambda t)$ .

If  $N_t$  is a Markov birth process with birth rates  $\lambda + n\mu$  then  $N_t$  is  $\text{NegBin}(\lambda t, \mu)$ .

# More Random Variables

Family	Range	Distribution	Notes	
Chi-squared	$\chi_n^2$	$\mathbb{R}^+$	$\chi_n^2 \sim \sum^n N(0, 1)^2$	$\chi_n^2 \sim \Gamma(\frac{n}{2}, \frac{1}{2})$
Student t	$t_n$	$\mathbb{R}$	$t_n \sim \frac{N(0, 1)}{\sqrt{\chi_n^2/n}}$	$t_1 \sim \text{Cauchy}$ . $\mathbb{E}t_n = 0$ if $n > 1$ ; $\text{Var } t_n = \frac{n}{n-2}$ if $n > 2$ .
F ratio	$F_{m,n}$	$\mathbb{R}^+$	$F_{m,n} \sim \frac{\chi_m^2/m}{\chi_n^2/n}$	$\mathbb{E}F_{m,n} = \frac{m}{m-2}$ for $m > 2$ .
Weibull	Weibull( $\alpha$ )	$\mathbb{R}^+$	$f(x) = \alpha x^{\alpha-1} e^{-x^\alpha}$	Weibull( $\alpha$ ) $\sim \text{Exp}(1)^{1/\alpha}$
Beta	Beta( $r, s$ )	$[0, 1]$	$f(x) = \frac{x^{r-1}(1-x)^{s-1}}{\beta(r, s)}$	Beta( $r, s$ ) $\sim \left(1 + \frac{\Gamma(s, \lambda)}{\Gamma(r, \lambda)}\right)^{-1}$ . Mean $\frac{r}{r+s}$ .
Logistic	Logistic	$\mathbb{R}$	$f(x) = \frac{e^x}{(1+e^x)^2}$	Logistic $\sim \log(\text{Pareto}(1) - 1)$ . $\mathbb{E}(e^{tX}) = \beta(1+t, 1-t)$ .
Pareto	Pareto( $\alpha$ )	$[1, \infty)$	$f(x) = \alpha x^{-(\alpha+1)}$	Pareto( $\alpha$ ) $\sim e^{\text{Exp}(\alpha)} \sim \text{Beta}(\alpha, 1)^{-1}$ . $\mathbb{E}X = \infty$ if $\alpha \leq 1$ ; $\text{Var } X = \infty$ if $\alpha \leq 2$ .

$$n! \sim \sqrt{2\pi n} n^{n+1/2} e^{-n}. \quad \Gamma(n) = (n-1)!. \quad \beta(r, s) = \frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)}.$$