Social and Technological Network Analysis

Lecture 5: Web Search and Random Walks

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In This Lecture

- We describe the concept of search in a network.
- We describe powerful techniques to enhance these searches.

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Search

- When searching “Computer Laboratory” on Google the first link is for the department’s page.

- How does Google know this is the best answer?
  - Information retrieval problem: synonyms (jump/leap), polysemy (Leopard), etc
  - Now with the web: diversity in authoring introduces issues of common criteria for ranking documents
  - The web offers abundance of information: whom do we trust as source?
- Still one issue: static content versus real time
  - World trade center query on 11/9/01
  - Twitter helps solving these issues these days

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Automate the Search

- We could collect a large sample of pages relevant to “computer laboratory” and collect their votes through their links.
- The pages receiving more in-links are ranked first.
- But if we use **the network structure** more deeply we can improve results.

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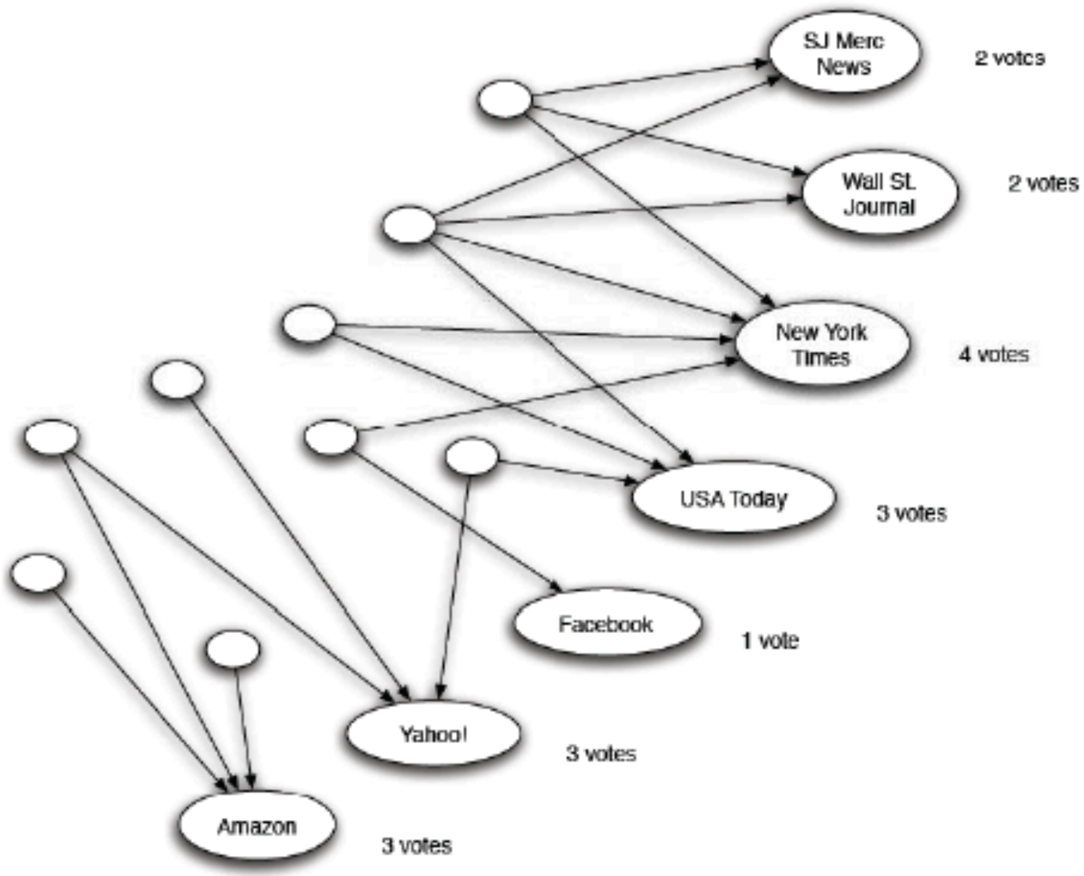
Example: Query "newspaper" Authorities

- Links are seen as votes.
- **Authorities** are established: the highly endorsed pages



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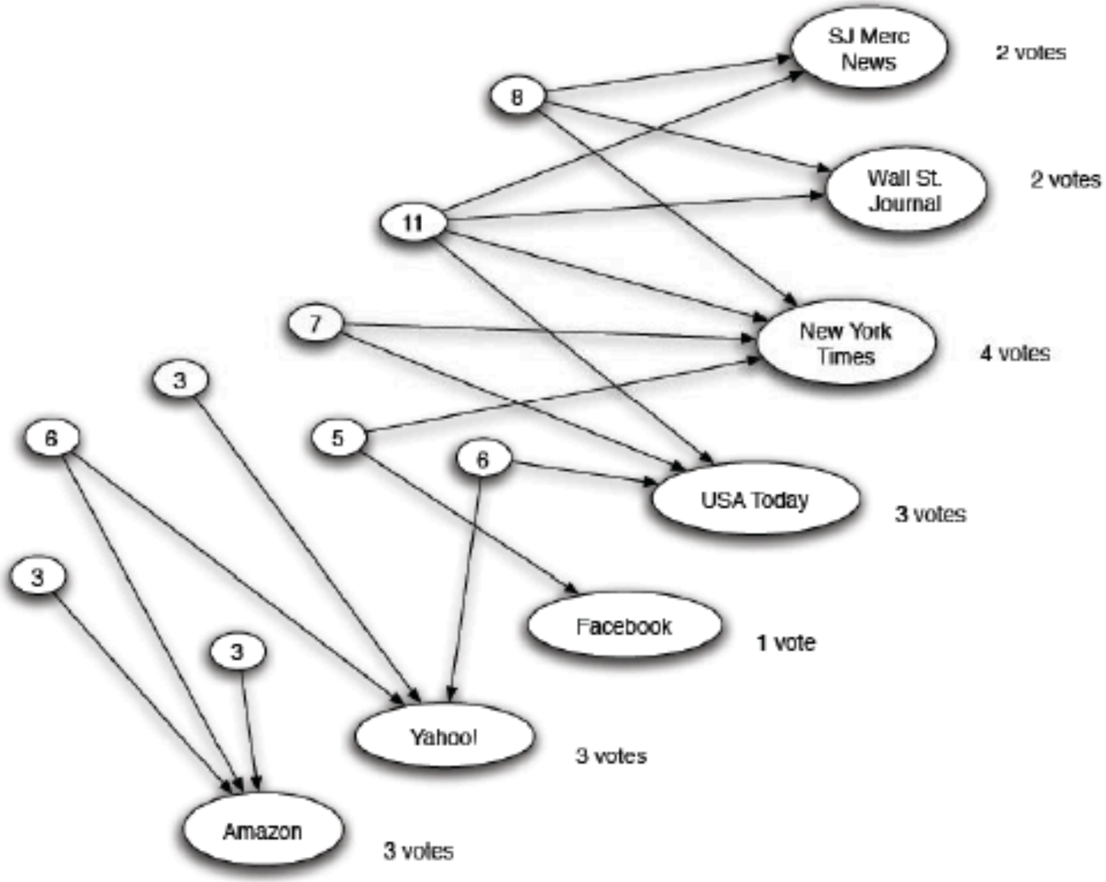
A Refinement: Hubs

- Numbers are reported back on the source page and aggregate.
- Hubs are high value lists



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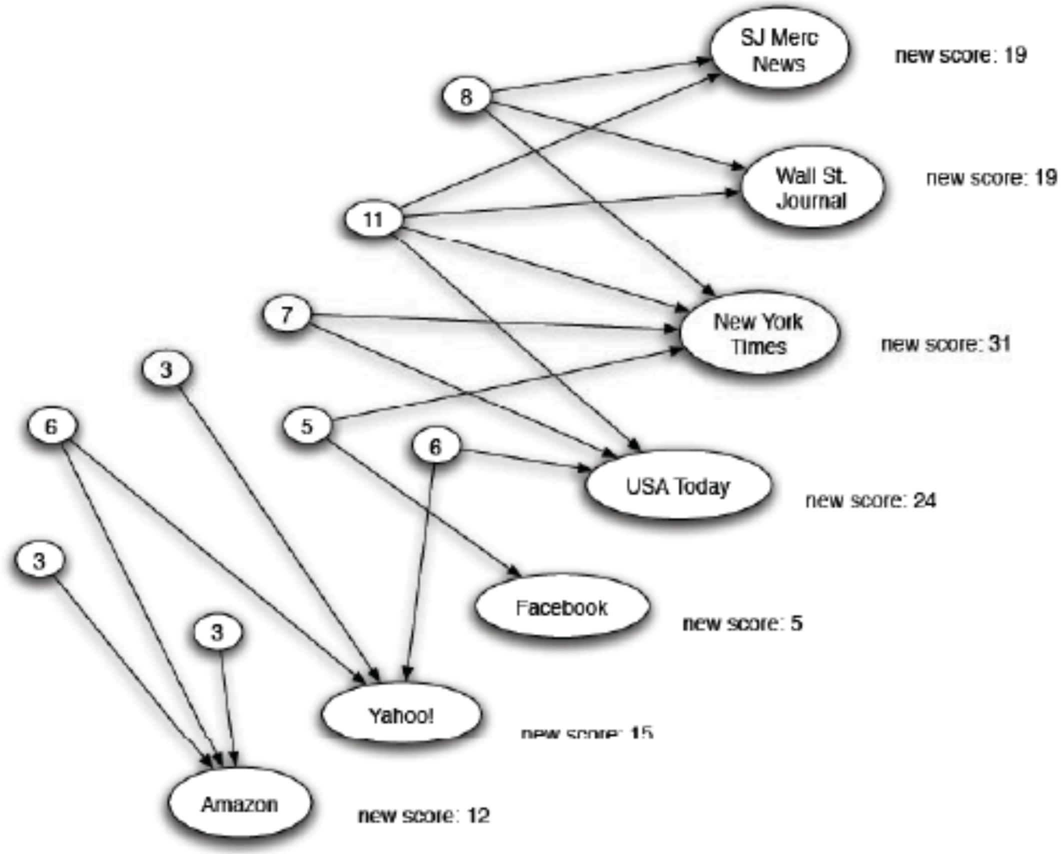
Principle of Repeated Improvement

- And we are now reweighting the authorities

- When do we stop?



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Repeating and Normalizing

- The process can be repeated
- Normalization:
  - Each authority score is divided by the sum of all authority scores
  - Each hub score is divided by the sum of all hub scores

More Formally: does the process converge?

- Each page has an authority a_i and a hub h_i score
- Initially $a_i = h_i = 1$
- At each step

$$a_i = \sum_{j->i} h_j$$

$$h_j = \sum_{j->i} a_i$$

- Normalize

$$\sum a_i = 1$$

$$\sum h_j = 1$$

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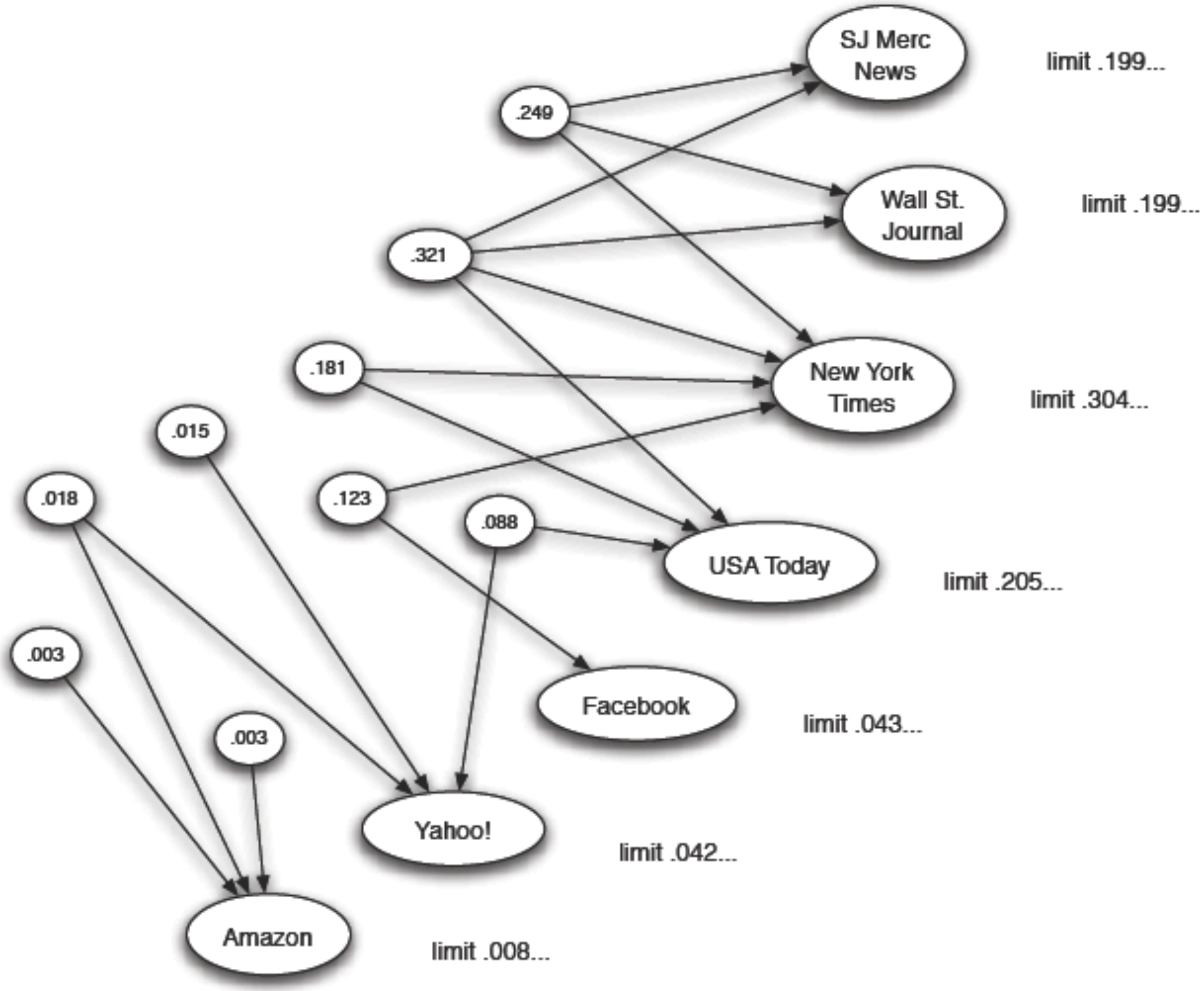
The process converges



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Let’s prove it

- Graph seen as matrix M_{ij} [where m_{ij}=1 if i->j]
- Vectors a=(a1,a2..an), h=(h1,h2,..hn)

$$h_i = \sum_{i->j} a_j \Leftrightarrow h_i = \sum_j M_{ij} a_j$$

- So $h=Ma$ and a=$M^T h$

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Algorithm

- Initially $a=h=1^n$
- Step
  - $h=Ma$, $a=M^Th$
  - Normalize
- Then $a=M^T(Ma)=(M^TM)a$
- And $h=M(M^Th)=(MM^T)h$

- In 2k steps
  $a=(M^TM)^k a$ and $h=(MM^T)^k h$

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Eigenvalues and eigenvectors

- **Definition:**
- *Let $Ax=\lambda x$ for scalar λ, vector x and matrix A*
- *Then x is an eigenvector and λ is its eigenvalue*

- If A is symmetric ($A^T=A$) then A has n orthogonal unit eigenvectors $w_1, \ldots w_n$ that form a basis (ie linear independent) with eigenvalues $\lambda_1,\ldots\lambda_n$ ($|\lambda_i| \leq |\lambda_{i+1}|$)
  - Tip: M^TM and MM^T are symmetric

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So…

- Given the linear independence of the w_i we can write $x=\Sigma p_i w_i$
- And x has coordinates [p1.. pn]
- If λ1,…λn (|λi|≤|λi+1|) then

- $A^k x=(\lambda_1^k p_1 w_1 + \lambda_2^k p_2 w_{2,\dots+} \lambda_n^k p_n w_n)=\Sigma_. \lambda_i^k p_i w_i$

- Let's divide by λ_1^k
  $A^k x/\lambda_1^k=\lambda_1^k p_1 w_{1/}\lambda_1^k + \lambda_2^k p_2 w_{2/}\lambda_1^k{}_{\dots+.} \lambda_n^k p_n w_{n/}\lambda_1^k$

- When k->∞, $A^k x/\lambda_1^k \to p_1 w_1$

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Convergence

- $(M^T M)^k a$ and $(M M^T)^k h$
- So for k->∞ these sequences converge

PageRank

- We have seen hubs and authorities
  - Hubs can "collect" links to important authorities who do not point to each others
  - There are other models: better for the web, where one prominent can endorse another.
- The **PageRank** model is based on transferrable importance.

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PageRank Concepts

- Pages pass endorsements on outgoing links as fractions which depend on out-degree
- Initial PageRank value of each node in a network of n nodes: 1/n.
- Choose a number of steps k.
- **[Basic] Update rule**: each page divides its pagerank equally over the outgoing links and passes an equal share to the pointed pages. Each page's new rank is the sum of received pageranks.

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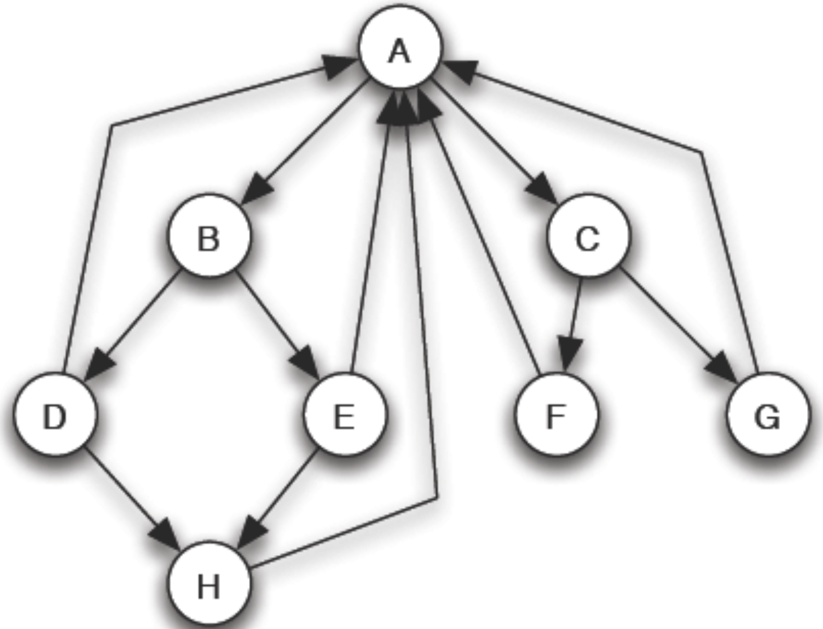
Example

- All pages start with PageRank= 1/8

Step	A	B	C	D	E	F	G	H
1	1/2	1/16	1/16	1/16	1/16	1/16	1/16	1/8
2	3/16	1/4	1/4	1/32	1/32	1/32	1/32	1/16

A becomes important and
B,C benefit too at step 2



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Convergence

- Except for some special cases, PageRank values of all nodes converge to limiting values when the number of steps goes to infinity.
- The convergence case is one where the PageRank of each page does not change anymore, i.e., they regenerate themselves.

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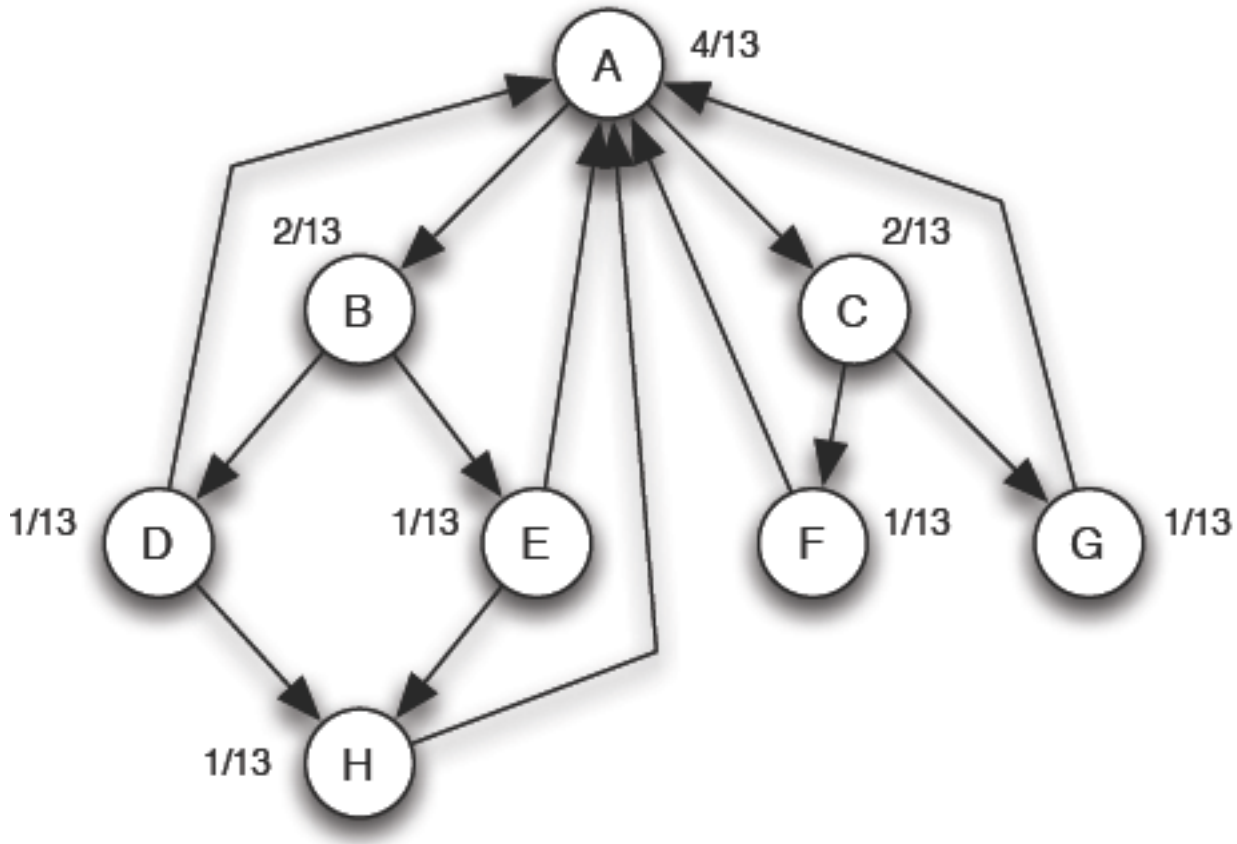
Example of Equilibrium



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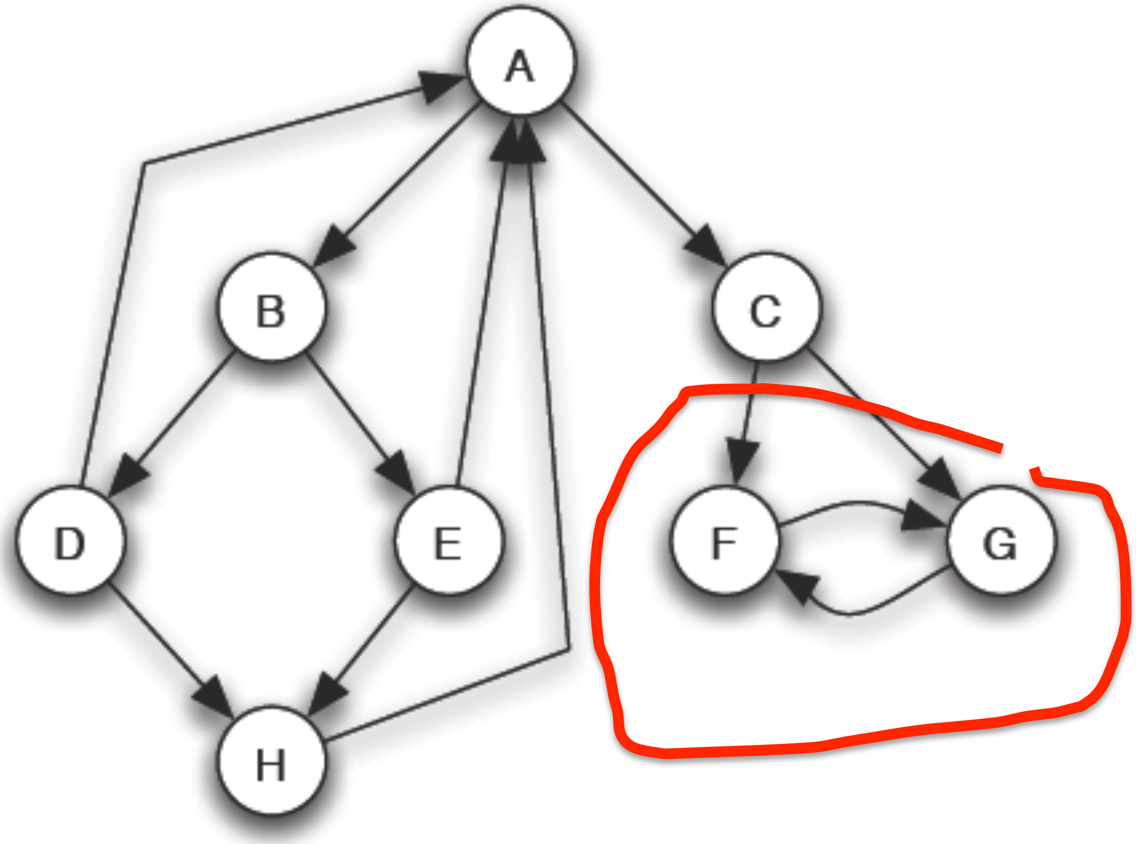
Problems with the basic PageRank
Dead ends

- F,G converge to ½ and all the other nodes to 0



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Solution: The REAL PageRank

- **[Scaled] Update Rule:**
  - Apply basic update rule. Then, scale down all values by scaling factor s [chosen between 0 and 1].
  - **[Total network PageRank value changes from 1 to s]**
  - Divide 1-s residual units of PageRank equally over all nodes: (1-s)/n each.

- It can be proven that values converge again.
- Scaling factor usually chosen between 0.8 and 0.9

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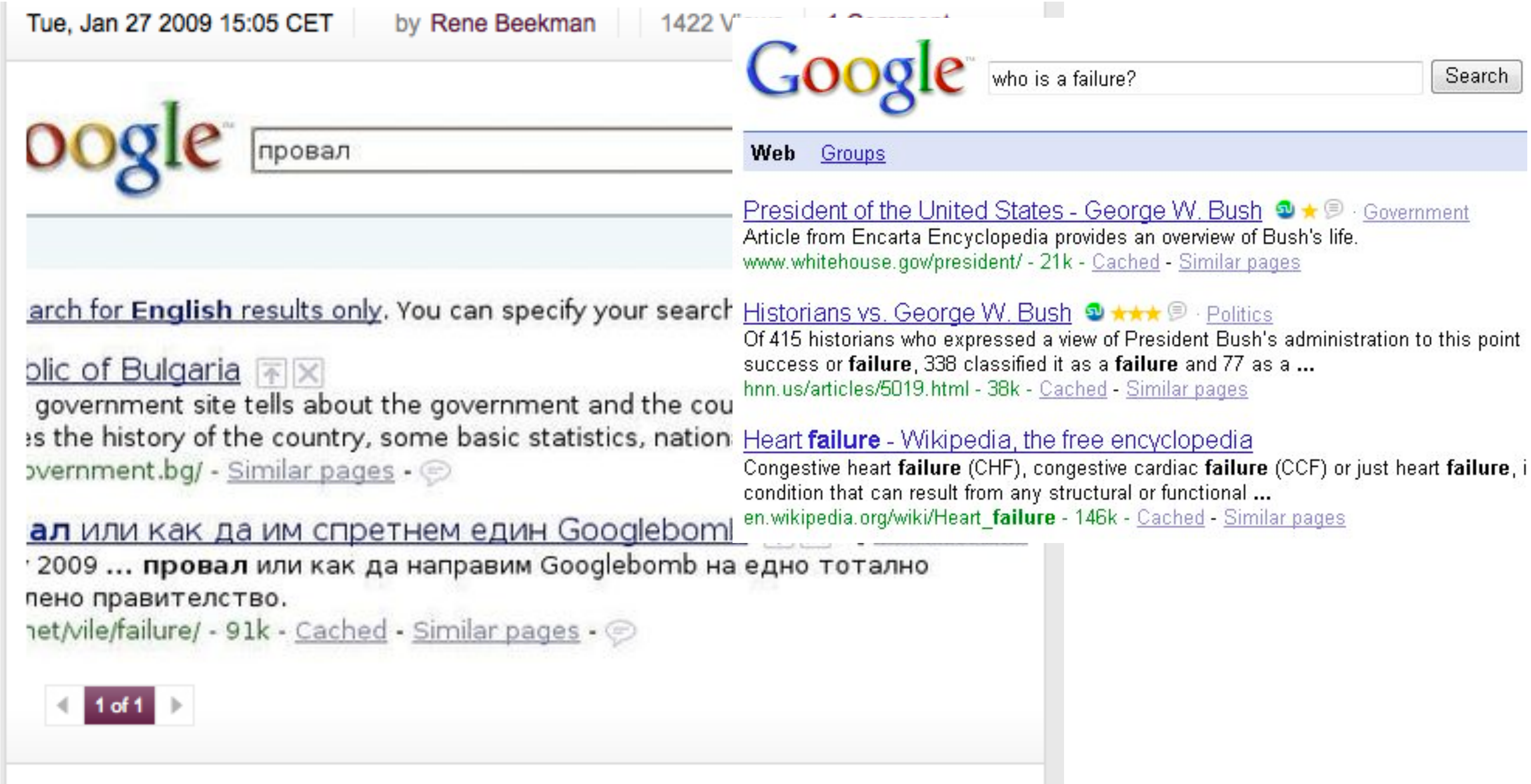
Search Ranking is very important to business

- A change in results in the search pages might mean loss of business
  - I.e., not appearing on first page.
- Ranking algorithms are kept very secret and changed continuously.

Examples of Google Bombs



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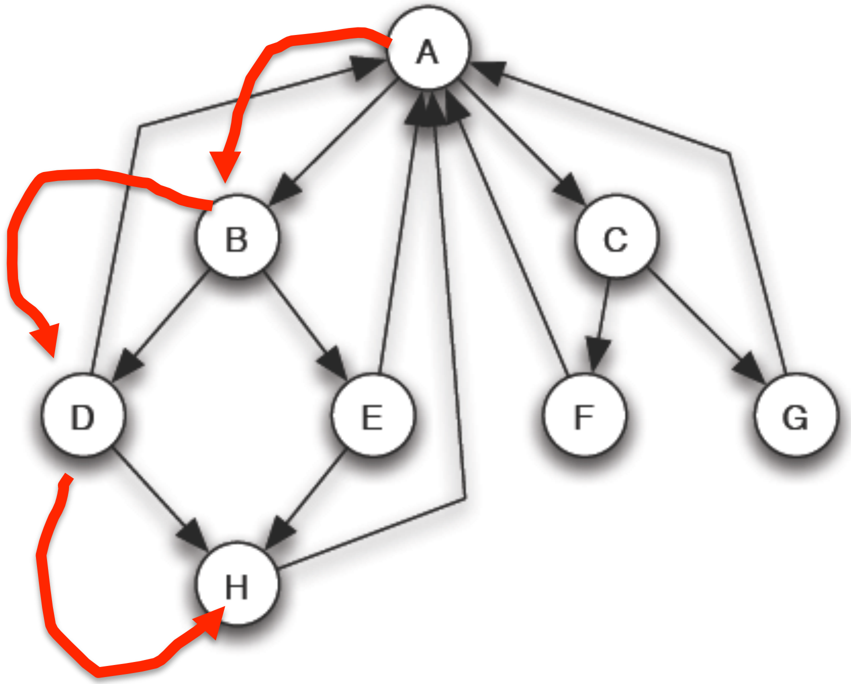
Random Walks

- Starting from a node, follow one outgoing link with random probability



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PageRank as Random Walk

- The probability of being at a page X after k steps of a random walk is precisely the PageRank of X after k applications of the Basic PageRank Update Rule
- Scaled Update Rule equivalent: follow a random outgoing link with probability s while with probability 1-s jump to a random node in the network.

References

- Chapter 14