# Probability Questions

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**Q1.** There are three cards. One has both faces white, one has both faces black, and the other has one face white and one face black. A card is chosen at random and its face is white. What is the probability that the other face is black?

**Q2.** In a town, all households have exactly two children. You meet a family that has a girl. What is the probability that she has a brother? How about if the girl is born on a Tuesday?

**Q3.** Three evenly matched chess players $A$, $B$, $C$ are in a tournament. $A$ plays $B$ first, and then the winner plays $C$. The winner always plays against the waiting player until a player wins twice in a row, becoming the champion. Find the probability that $A$ is the champion. What average number of games plays $C$ knowing that $A$ is the champion?

**Q4.** Are less, equal or more tosses of a Heads/Tails coin needed on average to get HTH as opposed to HTT? Justify analytically. Find the average number of tosses to get $M$ consecutive heads. How many tosses it takes on average to get one heads and two tails consecutively but in no particular order?

**Q5.** The real numbers $x$ and $y$ are chosen randomly in the $(0, 1)$ interval. Find the probability that the distance between $x$ and $y$ is greater than $x$.

**Q6.** The real numbers $x$ and $y$ are chosen randomly in the $(0, 1)$ interval. Find the probability that $\lfloor y/x \rfloor$ is an even number, where $\lfloor \cdot \rfloor$ is the floor function.

**Q7.** A stick is broken at two random points. Find the probability that the three pieces can form a triangle.

**Q8.** A stick is broken at one random point. Then, the longer resulting piece suffers the same fate. Find the probability that the three pieces can form a triangle.

**Q9.** A needle of length $l$ is dropped on a floor of infinitely long parallel tiles of width $w$. Find the probability that the needle falls on a tile edge when $l \leq w$ and also $l \geq w$.

**Q10.** An $m \times n$ grid has every cell painted at random red or blue. We say that two blocks A and B are in the same region if there is a sequence of blocks of the same colour starting at A and ending at B, successively adjacent to each other (regions of 1 block are valid). Find a lower bound for the average number of regions. *Hint: try first with* $m = 1$.

**Q11.** $N$ people board a plane. The first has lost his boarding pass and takes a random seat. Each subsequent passenger takes their assigned seat if available, or else a random free seat. Find the probability that the last passenger finds his seat occupied.

**Q12.** The real numbers $x, y, z, t$ are chosen randomly in the interval $(0, 1)$. Find the probability that the interval formed between $x$ and $y$ intersects the one between $z$ and $t$.

**Q13.** The real numbers $x, y, z, t$ are chosen randomly in the interval $(0, 1)$. Let $I_1$ be interval between $x$ and $y$, and $I_2$ between $z$ and $t$. Define $|I|$ as the length of interval $I$. Find the probability that $|I_1 \cup I_2| > \lambda \in (0, 1)$. Recall $|A \cup B| = |A| + |B| - |A \cap B|$.

**Q14.** The first $2n$ naturals are paired up at random to form $n$ intervals. Find the probability there is at least one interval that intersects all others (which we call a *super interval*).

**Q15.** *Extension*. Find the probability that there are exactly $k$ super intervals, $k = 0, \ldots, n$.

**Q16.** $2n$ reals are randomly chosen in $[0, 1]$ and then paired to form $n$ intervals. Find the probability that their intersection is not empty.

**Q17.** How many throws of one die are needed on average until all 6 numbers show? How many throws until each number shows at least twice?

**Q18.** How many coin tosses are needed on average until the number of heads equals the number of tails?

**Q19.** How many coin tosses are needed on average until you get $n$ heads, not necessarily consecutively?

**Q20.** What is the probability that $k$ consecutive tails never occurs in $n$ tosses of a coin?

**Q21.** What is the probability that $k$ consecutive tails occurs for the first time at the $n$-th toss? Use the answer to find the average number of tosses needed to get $M$ consecutive tails.

**Q22.** Three points are taken at random on an infinite plane. Find the probability that they form an obtuse triangle. In other words, are most triangles obtuse?

**Q23.** How many coin tosses are needed on average until you get an odd number of heads?

**Q24.** How many throws of one die are needed on average until each face appears at least $n$ times?

**Q25.** The real numbers $x$ and $y$ are chosen randomly in the $(0, 1)$ interval. Find the probability density function of the distance between $x$ and $y$.

**Q26.** A stick is broken at two random points. Find the probability density function of the longest piece.

**Q27.** A stick is broken at one random point. Then, the longer resulting piece suffers the same fate. Find the probability density function of the longest piece.