Irrotational motion of a compressible inviscid fluid

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Introduction

Compressible inviscid fluid
eg air with no viscosity or thermal conductivity

- Equations by Leonhard Euler 1707-1783
- Taught to undergraduates
  – No special knowledge needed
- All equations completely classical
  – no quantum mechanics

Euler's equation for a compressible fluid

Euler's equation describes the air without viscosity

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla P$$

Reduces to wave equation at low amplitude

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = 0$$

where $c^2 = \partial P / \partial \rho$

Rotational solutions

http://www.youtube.com/watch?v=bT-fctr32pE

- Rotational - defined by $\oint u.dl \neq 0$
- We will examine an irrotational equivalent of these

Irrotational motion of a compressible inviscid fluid

Solutions to **wave** equation (low amplitude)

$$\Delta\rho = A\cos(\omega_o t + m\theta)\, J_m(k_r r)$$

(J_m – cylindrical Bessel function)

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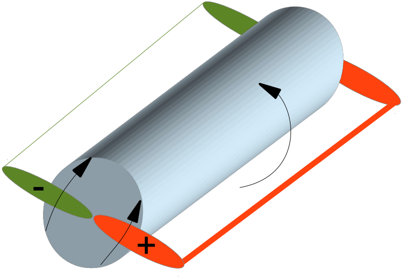

(animation)

m = 1
Irrotational – $\oint u.dl = 0$

Sonon quasiparticles

Curve the eddy into a ring

- Quasiparticle solution similar to Dolphin air ring – sonon

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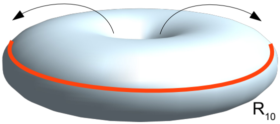

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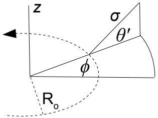

$$\xi = A\, e^{-i\omega_o t}\, R_{mn}(\mathbf{x})$$

$$R_{mn}(\mathbf{x}) = \oint e^{-i(m\theta' - n\phi)} j_m(k_r \sigma)\, R_o d\phi$$

j_m – spherical Bessel function

Solution for $m = 0, 1$ (need Legendre polynomials for $m > 1$)

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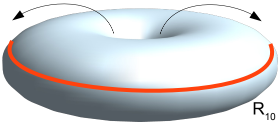

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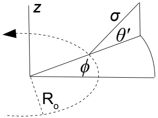

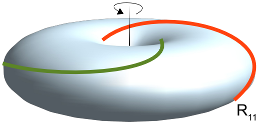

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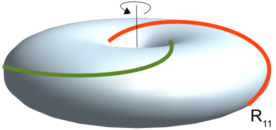
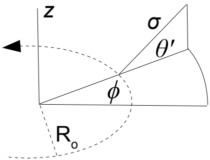


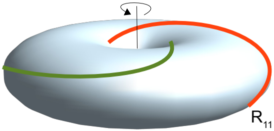


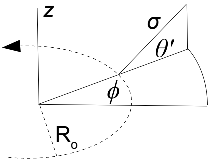
$\oint e^{-i(m\theta' - n\phi)} j_m(k_r \sigma)\ R_o d\phi$

factorise at large r

$[B_r.\Phi_r(r)]\ [B_\phi.\Phi_\phi(\phi)]\ [B_\theta.\Phi_\theta(\theta)]$

Density pattern at large r





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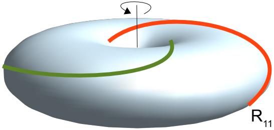
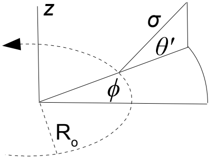
$$[B_r.\Phi_r(r)]\ [B_\phi.\Phi_\phi(\phi)]\ [B_\theta.\Phi_\theta(\theta)]$$

$$\Phi_r = \frac{1}{r}[\sin(k_r r), \cos(k_r r)]$$

$$\Phi_\phi = [e^{-in\phi}, e^{in\phi}]$$

$$\Phi_\theta = [e^{-im\theta} + e^{im\theta}, e^{-im\theta} - e^{im\theta}]$$





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$$\Phi_r = \frac{1}{r}[\sin(k_r r), \cos(k_r r)]$$

$$i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.\frac{\partial\Phi}{\partial r} = k_r\Phi$$

$$\Phi_\phi = [e^{-in\phi}, e^{in\phi}]$$

$$i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\frac{\partial\chi}{\partial\Phi} = n\Phi$$

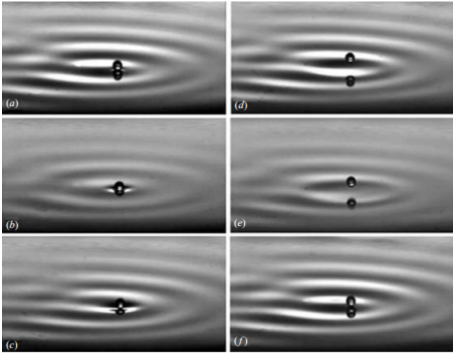
$$\Phi_\theta = [e^{-im\theta} + e^{im\theta}, e^{-im\theta} - e^{im\theta}]$$

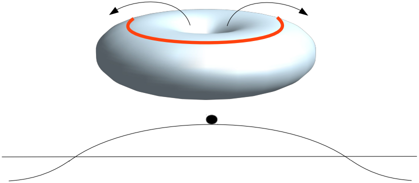
$$i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.\frac{\partial\Phi}{\partial\theta} = m\Phi$$

Pauli spin matrix

Irrotational motion of a compressible inviscid fluid

Experimental analogue in 2D





(particle drawn just for illustration)

Droplet bouncing on the surface of the same liquid

(Bath vibrated vertically)

http://www.youtube.com/watch?v=B9AKCJjtKa4

S Protiere, A Badaoud, Y Couder Particle wave association on a fluid interface J Fluid Mech 544 85-108 (2006)

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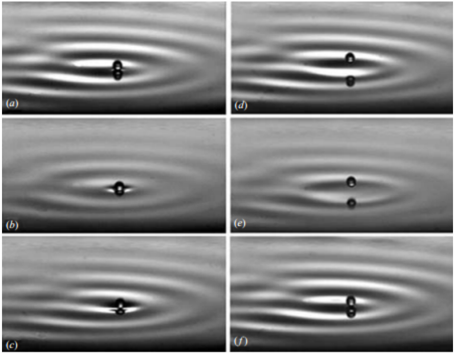

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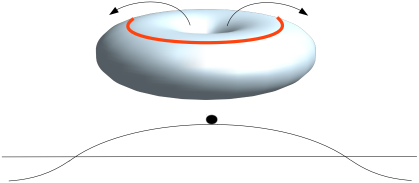

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Main difference from sonons

- 2D driven dissipative system
- Effective *c* reduced near droplet
- Droplet itself does not obey Euler's equation

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Lorentz covariance

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$A \ll 1$

- Obeys the wave equation
- The wave equation is unchanged by Lorentz transformation
- If $\xi(x, t)$ is a solution then so is $\xi(x', t')$
- $\xi(x', t')$ moves at velocity v, Lorentz-Fitzgerald contraction

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Finite amplitude

- perturbed by $\epsilon = (v.\nabla)u$ if moving at constant velocity v
- but the sonon is oscillatory: $\int udt = 0$ over a cycle
- Therefore $\int \epsilon dt = 0$, ie perturbation vanishes over a cycle

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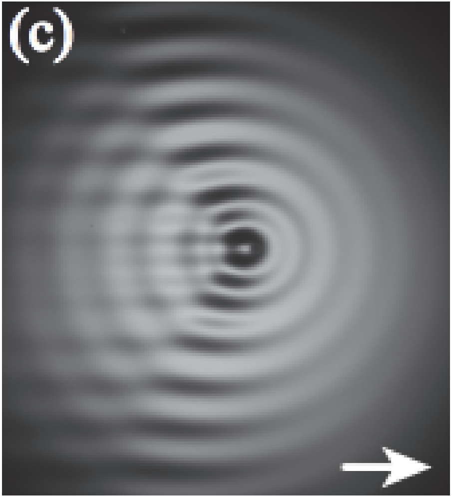
Expectation values converge on long term average

- Lorentz covariant at all amplitudes

Experimental analogue - walker



'Walker'

Increase amplitude

- Droplet bounces higher
- Frequency reduces
- Velocity v increases

$$v = c'\sqrt{\frac{A - A_o}{A}} \text{ where } c' < c$$

A Eddi et al 'Information stored in Faraday waves: the origin of a path memory' J Fluid Mech. 674 433-463 (2011)

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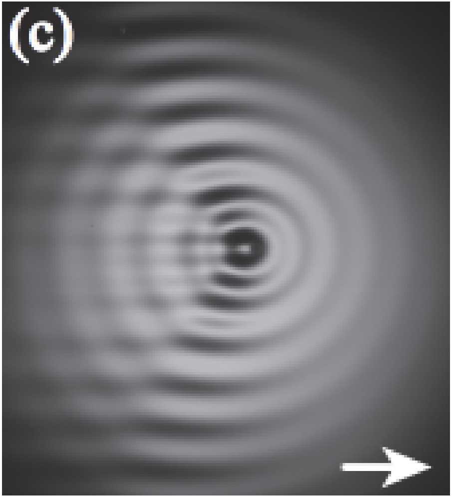

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Rearrange

- Approximate $\tau \propto \sqrt{A}$

$\omega \approx \frac{\omega_o}{\gamma}$ where $\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c'^2}}$

Time dilation

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Equation for ψ

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Must be Lorentz covariant

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi = -\omega_o^2 \psi$$

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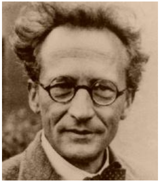
Klein-Gordon equation

Low velocity

$\psi = exp(-i\omega_o t)\psi'$, neglect $\frac{\partial^2}{\partial t^2}\psi'$

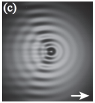
$$i\hbar\frac{\partial}{\partial t}\psi' + \frac{\hbar^2}{2m}\nabla^2\psi' = V\psi'$$



Schrödinger equation

units where $m = \hbar\omega_o/c^2$

Equation of motion for particle



Quasiparticle aligned with wave troughs (n.b. easily disrupted)

Speed v of the wave troughs $k \quad = \quad \frac{\omega}{c^2} v$

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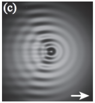

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1952 - Bohm rearranged Schrödinger's

$$\frac{\partial |\psi|^2}{\partial t} = -\nabla (|\psi|^2 v)$$

Continuity equation for $|\psi|^2$

$|\psi(\mathbf{x}, t)|^2$ = probability of reaching $(\mathbf{x}, t)$ (averaged over nearby trajectories)

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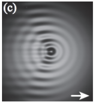

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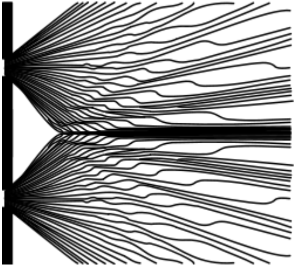

Bohmian trajectories

Aditional (superfluous) assumption

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  – same equations for ψ and collapse
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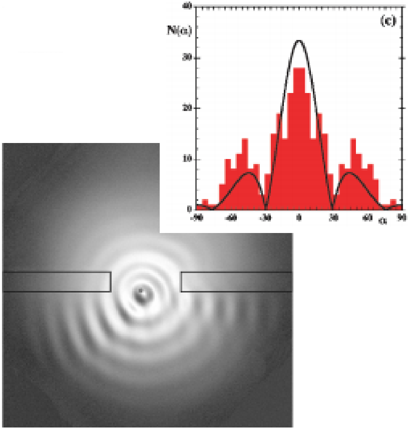
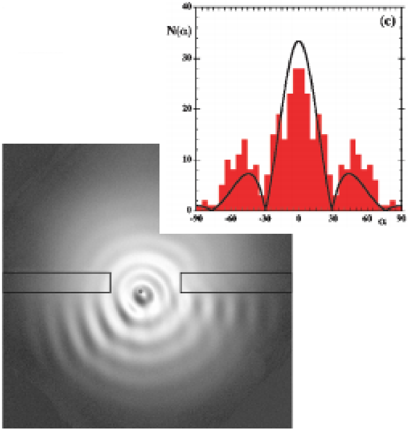
Reminder
– Euler's equation and all sonon motion is completely classical

Classical experiment – diffraction



Single slit

Y Couder, E Fort ‘Single-Particle Diffraction and Interference at a Macroscopic Scale’ PRL 97 154101 (2006)



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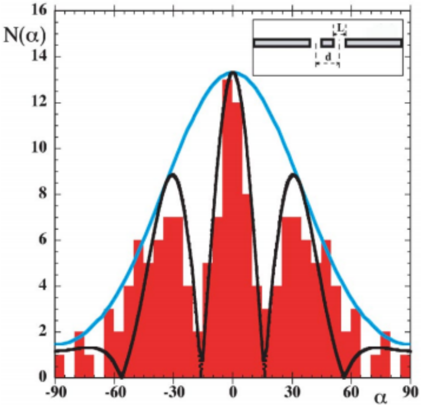

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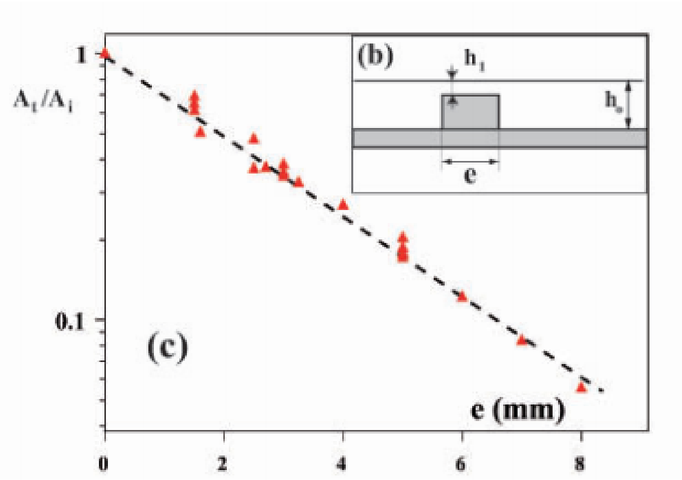
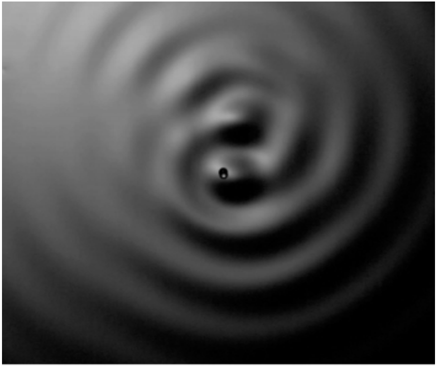
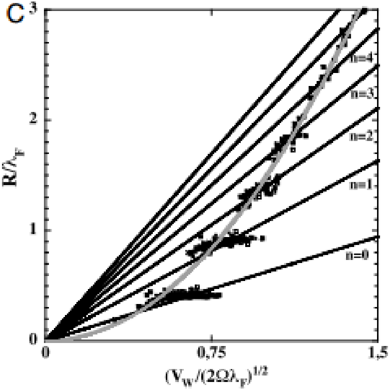
Classical experiment – tunnelling



A Eddi, E Fort, F Moisi, Y Couder 'Unpredictable tunneling of a classical wave-particle association' PRL 102, 240401 (2009)

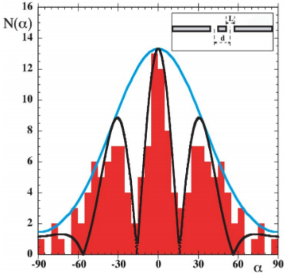
Classical experiment – Landau levels

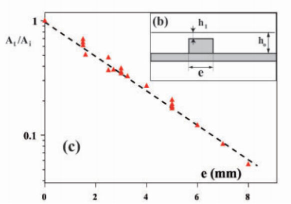


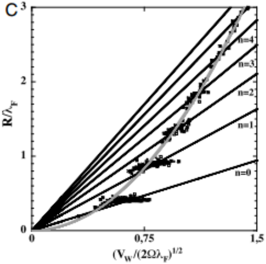


E Fort et al ‘Path-memory induced quantization of classical orbits’ PNAS 107 41 17515-17520 (2010)

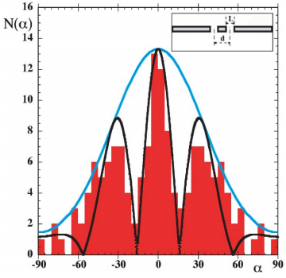
Classical experiment – incapable of quantum collapse

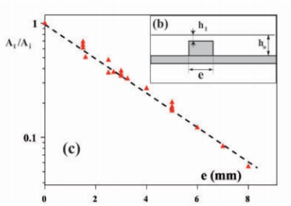


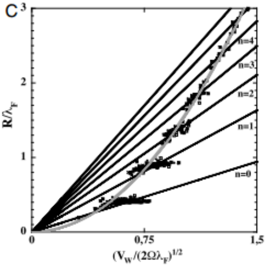




Wavefunction collapse **superfluous**







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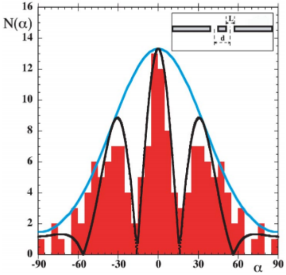

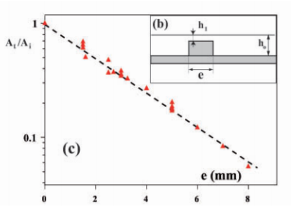

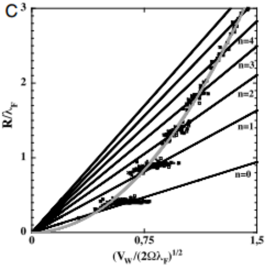

$$\xi \quad = \quad A\, e^{-i(\omega t - kx)} \quad R_{mn}(\mathbf{x}')$$
$$= \quad \psi \quad \chi$$

ψ obeys same equations as quantum mechanical wavefunction
ψ modulates χ (usually omitted) – localisation

Classical experiment – incapable of quantum collapse

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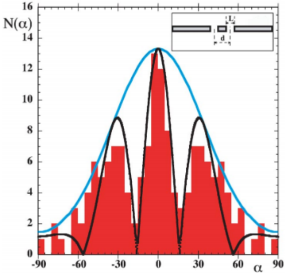

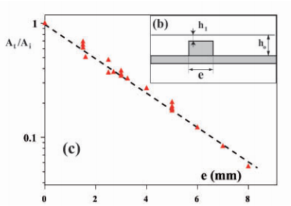

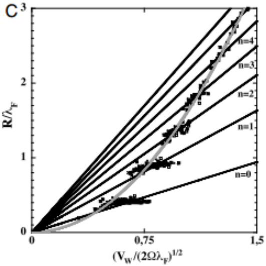

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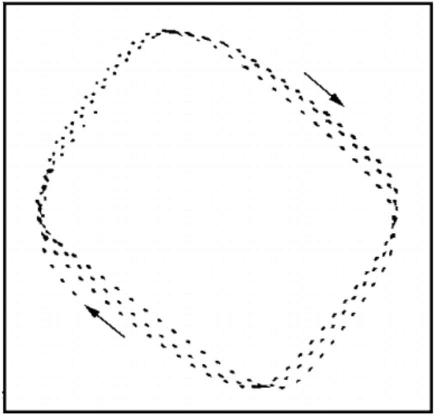
modulation carrier

Modulation of a carrier wave
amplitude and phase – complex valued

ψ obeys same equations as quantum mechanical wavefunction

ψ modulates χ (usually omitted) – localisation

Forces between quasiparticles – 2D



Stroboscopic photograph

- Meniscus at boundary
- Image droplet antiphase
- Repulsive force

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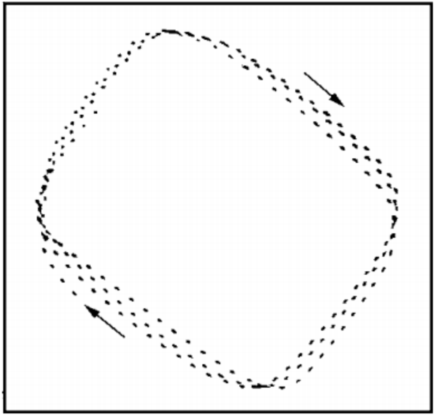

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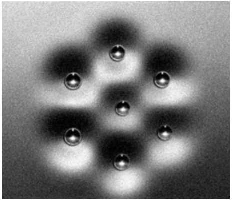

In-phase attraction

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Forces between quasiparticles – 3D

Ultrasonic degassing of oil
(5 seconds)

- Switch on ultrasonic transducer
- Bubbles expand and contract in phase
- Fluid dynamic attraction
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- Opposite chiralities attract
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Magnitude of the force

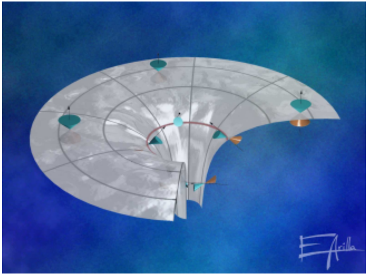
- Characterised by dimensionless number $\alpha \lesssim \frac{1}{49}$
- Could be calculated more accurately by computer simulation
- Experimental fine structure constant $\alpha = \frac{1}{137.035999074}$

Irrotational motion of a compressible inviscid fluid

Extends field of 'analogue gravity'

Irrotational motion of a compressible inviscid fluid

- 'Acoustic metric' like general relativity
- 1981 Unruh proposed sonic experiment: Hawking radiation



Acoustic black hole

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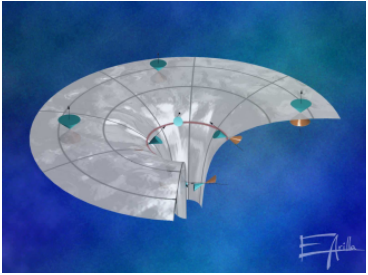

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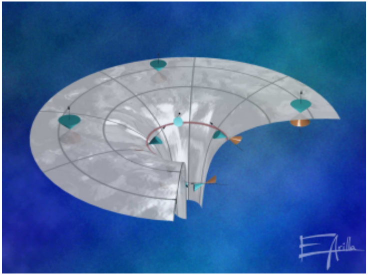

Acoustic black hole

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Possible interpretation

Why do gyroscopes remain aligned with the fixed stars?

Superconducting gyroscope
(and me with hair)

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- All interactions are local
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Euclidian space, compressible fluid

- Spin-$\frac{1}{2}$ quasiparticles
- Obey special relativity
- Diffract like quantum particles
- Electromagnetic force between them

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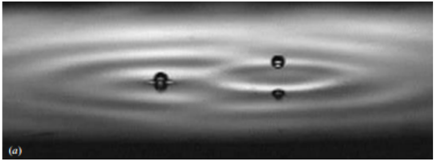
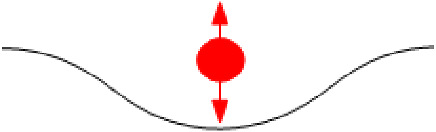
Irrotational motion of a compressible inviscid fluid

Limits to coherence



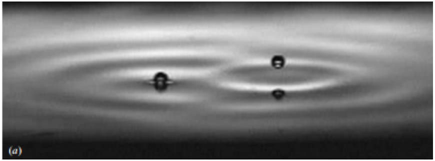
Coherence between particles



needs coherence with carrier wave

One correlation per dimension = 3 independent correlations

Limits to coherence



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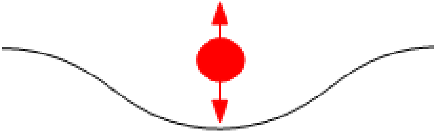

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Quantum computing
– major multi-year research effort

- Relies on multiple correlations (or possibly entanglement)

R Anderson, R Brady 'Why quantum computing is hard' in preparation

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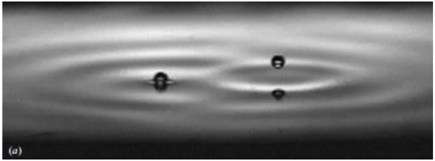

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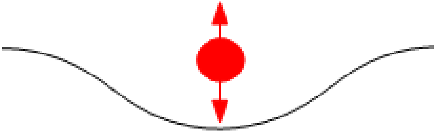

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Quantum computing
– major multi-year research effort

- Relies on multiple correlations (or possibly entanglement)
- Proxies for qubits, but no calculations with > 3 qubits

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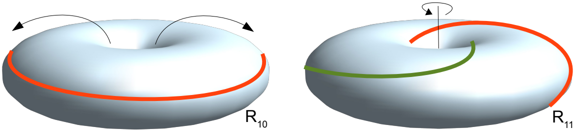
Irrotational motion of a compressible inviscid fluid

Irrotational solutions to Euler's equation



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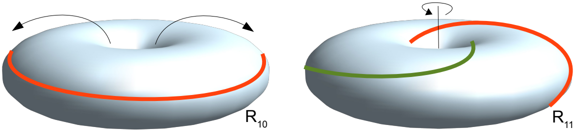

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- Move and diffract like quantum particles of mass $\hbar\omega/c^2$
- Maxwell's equations with $\alpha \lesssim 1/45$

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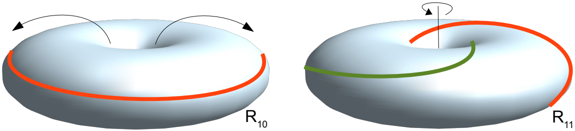

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Just ordinary Newton's equations applied to a fluid

- No wavefunction collapse, multiverses, cats or dice
- No distortion of space and time
  – Lorentz covariance a symmetry of Euler's equation
- No action at a distance – ordinary forces mediated by the fluid