

<i>typevar</i> , X	type variable		
<i>termvar</i> , x	term variable		
<i>label</i> , l, k	field label		
<i>index</i> , i, j, n, m	indices		
T, S, U	$::=$		type
	X		type variable
	Top		maximum type
	$T \rightarrow T'$		type of functions
	$\forall X <: T. T'$	bind X in T'	universal type
	$\{ l_1 : T_1, \dots, l_n : T_n \}$		record
	(T)	M	
	$[X \mapsto T] T'$	M	
t	$::=$		term
	x		variable
	$\lambda x:T. t$	bind x in t	abstraction
	$t t'$		application
	$\Lambda X <: T. t$	bind X in t	type abstraction
	$t [T]$		type application
	$\{ l_1 = t_1, \dots, l_n = t_n \}$		record
	$t.l$		projection
	let $p = t$ in t'	bind $b(p)$ in t'	pattern binding
	(t)	M	
	$[x \mapsto t] t'$	M	
	$[X \mapsto T] t$	M	
	σt	M	
p	$::=$		pattern
	$x : T$	$b = x$	variable pattern
	$\{ l_1 = p_1, \dots, l_n = p_n \}$	$b = b(p_1..p_n)$	record pattern
v	$::=$		values
	$\lambda x:T. t$	bind x in t	abstraction
	$\Lambda X <: T. t$	bind X in t	type abstraction
	$\{ l_1 = v_1, \dots, l_n = v_n \}$		record
Γ, Δ	$::=$		type environment
	empty		
	$\Gamma, X <: T$		
	$\Gamma, x : T$		
	$\Gamma_1, \dots, \Gamma_n$	M	
σ	$::=$		multiple term substitution
	$[x \mapsto t]$		
	$\sigma_1, \dots, \sigma_n$		
<i>terminals</i>	$::=$		
	λ		
	$\rightarrow$		
	$\Rightarrow$		
	$\vdash$		

		$\longrightarrow$	
		$\forall$	
		$<:$	
		$\vdash$	
		$\wedge$	
		$\vee$	
		$=$	
<i>formula</i>	::=		
		<i>judgement</i>	
		$x = x'$	
		$X = X'$	
		$(\textit{formula})$	
		$\neg \textit{formula}$	
		$\forall i \in 1..m. \textit{formula}$	
		$\exists i \in 1..m. \textit{formula}$	
		$\textit{formula} \wedge \textit{formula}'$	
		$l = l'$	
		$\textit{formula}_1 \dots \textit{formula}_n$	
<i>Judgement_in</i>	::=		
		$x \in \mathbf{dom}(\Gamma)$	
		$X \in \mathbf{dom}(\Gamma)$	
		$x : T \in \Gamma$	
		$X <: U \in \Gamma$	
<i>Jtype</i>	::=		
		$\Gamma \vdash \mathbf{ok}$	type environment Γ is well-formed
		$\Gamma \vdash T$	type T is well-formed in type environment Γ
		$\Gamma \vdash S <: T$	S is a subtype of T
		$\Gamma \vdash t : T$	term t has type T
		$\vdash p : T \Rightarrow \Delta$	pattern p matches type T giving bindings Δ
<i>Jop</i>	::=		
		$t_1 \longrightarrow t_2$	t_1 reduces to t_2
		$\mathbf{match}(p, v) = \sigma$	
<i>judgement</i>	::=		
		<i>Judgement_in</i>	
		<i>Jtype</i>	
		<i>Jop</i>	
<i>user_syntax</i>	::=		
		<i>typevar</i>	
		<i>termvar</i>	
		<i>label</i>	
		<i>index</i>	

	T
	t
	p
	v
	Γ
	σ
	<i>terminals</i>
	<i>formula</i>

$$\boxed{x \in \mathbf{dom}(\Gamma)}$$

$$\overline{x \in \mathbf{dom}(\Gamma, x : T)} \quad \text{XING_1}$$

$$\frac{x \in \mathbf{dom}(\Gamma)}{x \in \mathbf{dom}(\Gamma, X' <: U')} \quad \text{XING_2}$$

$$\frac{x \in \mathbf{dom}(\Gamma)}{x \in \mathbf{dom}(\Gamma, x' : T')} \quad \text{XING_3}$$

$$\boxed{X \in \mathbf{dom}(\Gamma)}$$

$$\overline{X \in \mathbf{dom}(\Gamma, X <: U)} \quad \text{XING_1}$$

$$\frac{X \in \mathbf{dom}(\Gamma)}{X \in \mathbf{dom}(\Gamma, X' <: U')} \quad \text{XING_2}$$

$$\frac{X \in \mathbf{dom}(\Gamma)}{X \in \mathbf{dom}(\Gamma, x' : T')} \quad \text{XING_3}$$

$$\boxed{x : T \in \Gamma}$$

$$\overline{x : T \in \Gamma, x : T} \quad \text{TIN_1}$$

$$\frac{x : T \in \Gamma}{x : T \in \Gamma, X' <: U'} \quad \text{TIN_2}$$

$$\frac{x : T \in \Gamma}{x : T \in \Gamma, x' : T'} \quad \text{TIN_3}$$

$$\boxed{X <: U \in \Gamma}$$

$$\overline{X <: U \in \Gamma, X <: U} \quad \text{TIN_1}$$

$$\frac{X <: U \in \Gamma}{X <: U \in \Gamma, X' <: U'} \quad \text{TIN_2}$$

$$\frac{X <: U \in \Gamma}{X <: U \in \Gamma, x' : T'} \quad \text{TIN_3}$$

$$\boxed{\Gamma \vdash \mathbf{ok}} \quad \text{type environment } \Gamma \text{ is well-formed}$$

$$\overline{\mathbf{empty} \vdash \mathbf{ok}} \quad \text{GOK_1}$$

$$\frac{\Gamma \vdash T \quad \neg(x \in \mathbf{dom}(\Gamma))}{\Gamma, x : T \vdash \mathbf{ok}} \quad \text{GOK_2}$$

$$\frac{\Gamma \vdash T \quad \neg(X \in \mathbf{dom}(\Gamma))}{\Gamma, X <: T \vdash \mathbf{ok}} \quad \text{GOK_3}$$

$\boxed{\Gamma \vdash T}$ type T is well-formed in type environment Γ

$$\frac{\Gamma \vdash \mathbf{ok} \quad X <: U \in \Gamma}{\Gamma \vdash X} \quad \text{GT_VAR}$$

$$\frac{\Gamma \vdash \mathbf{ok}}{\Gamma \vdash \mathbf{Top}} \quad \text{GT_TOP}$$

$$\frac{\Gamma \vdash T \quad \Gamma \vdash T'}{\Gamma \vdash T \rightarrow T'} \quad \text{GT_FUN}$$

$$\frac{\Gamma, X <: T \vdash T'}{\Gamma \vdash \forall X <: T. T'} \quad \text{GT_FORALL}$$

$$\frac{\Gamma \vdash T_1 \quad \dots \quad \Gamma \vdash T_n}{\Gamma \vdash \{l_1 : T_1, \dots, l_n : T_n\}} \quad \text{GT_RCD}$$

$\boxed{\Gamma \vdash S <: T}$ S is a subtype of T

$$\frac{\Gamma \vdash \mathbf{ok}}{\Gamma \vdash S <: \mathbf{Top}} \quad \text{SA_TOP}$$

$$\frac{\Gamma \vdash \mathbf{ok}}{\Gamma \vdash X <: X} \quad \text{SA_REFL_TVAR}$$

$$\frac{X <: U \in \Gamma \quad \Gamma \vdash U <: T}{\Gamma \vdash X <: T} \quad \text{SA_TRANS_TVAR}$$

$$\frac{\Gamma \vdash T_1 <: S_1 \quad \Gamma \vdash S_2 <: T_2}{\Gamma \vdash S_1 \rightarrow S_2 <: T_1 \rightarrow T_2} \quad \text{SA_ARROW}$$

$$\frac{\Gamma \vdash T_1 <: S_1 \quad \Gamma, X <: T_1 \vdash S_2 <: T_2}{\Gamma \vdash \forall X <: S_1. S_2 <: \forall X <: T_1. T_2} \quad \text{SA_ALL}$$

$$\frac{\forall i \in 1..m. \exists j \in 1..n. (k_i = l_j \wedge \Gamma \vdash S_i <: T_j)}{\Gamma \vdash \{k_1 : S_1, \dots, k_m : S_m\} <: \{l_1 : T_1, \dots, l_n : T_n\}} \quad \text{SA_RCD}$$

$\boxed{\Gamma \vdash t : T}$ term t has type T

$$\frac{\Gamma \vdash \mathbf{ok} \quad x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{TY_VAR}$$

$$\frac{\Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2} \quad \text{TY_ABS}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 : T_{11} \rightarrow T_{12} \quad \Gamma \vdash t_2 : T_{11}}{\Gamma \vdash t_1 t_2 : T_{12}} \quad \text{TY_APP} \\
\\
\frac{\Gamma, X <: T_1 \vdash t_2 : T_2}{\Gamma \vdash \Lambda X <: T_1. t_2 : \forall X <: T_1. T_2} \quad \text{TY_TABS} \\
\\
\frac{\Gamma \vdash t_1 : \forall X <: T_{11}. T_{12} \quad \Gamma \vdash T_2 <: T_{11}}{\Gamma \vdash t_1 [T_2] : [X \mapsto T_2] T_{12}} \quad \text{TY_TAPP} \\
\\
\frac{\Gamma \vdash t_1 : T_1 \quad \vdash p : T_1 \Rightarrow \Delta \quad \Gamma, \Delta \vdash t_2 : T_2}{\Gamma \vdash \text{let } p = t_1 \text{ in } t_2 : T_2} \quad \text{TY_LET} \\
\\
\frac{\Gamma \vdash t_1 : T_1 \quad \dots \quad \Gamma \vdash t_n : T_n}{\Gamma \vdash \{l_1 = t_1, \dots, l_n = t_n\} : \{l_1 : T_1, \dots, l_n : T_n\}} \quad \text{TY_RCD} \\
\\
\frac{\Gamma \vdash t : \{l_1 : T_1, \dots, l_n : T_n\}}{\Gamma \vdash t.l_j : T_j} \quad \text{TY_PROJ} \\
\\
\frac{\Gamma \vdash t : S \quad \Gamma \vdash S <: T}{\Gamma \vdash t : T} \quad \text{TY_SUB} \\
\\
\boxed{\vdash p : T \Rightarrow \Delta} \quad \text{pattern } p \text{ matches type } T \text{ giving bindings } \Delta \\
\\
\frac{}{\vdash x : T : T \Rightarrow \text{empty}, x : T} \quad \text{PAT_VAR} \\
\\
\frac{\vdash p_1 : T_1 \Rightarrow \Delta_1 \quad \dots \quad \vdash p_n : T_n \Rightarrow \Delta_n}{\vdash \{l_1 = p_1, \dots, l_n = p_n\} : \{l_1 : T_1, \dots, l_n : T_n\} \Rightarrow \Delta_1, \dots, \Delta_n} \quad \text{PAT_RCD} \\
\\
\boxed{t_1 \longrightarrow t_2} \quad t_1 \text{ reduces to } t_2 \\
\\
\frac{}{(\lambda x : T_{11}. t_{12}) v_2 \longrightarrow [x \mapsto v_2] t_{12}} \quad \text{REDUCE_APPABS} \\
\\
\frac{}{(\Lambda X <: T_{11}. t_{12}) [T_2] \longrightarrow [X \mapsto T_2] t_{12}} \quad \text{REDUCE_TAPPTABS} \\
\\
\frac{\text{match}(p, v_1) = \sigma}{\text{let } p = v_1 \text{ in } t_2 \longrightarrow \sigma t_2} \quad \text{REDUCE_LETV} \\
\\
\frac{}{\{l'_1 = v_1, \dots, l'_n = v_n\}. l'_j \longrightarrow v_j} \quad \text{REDUCE_PROJRCD} \\
\\
\frac{t_1 \longrightarrow t'_1}{t_1 t \longrightarrow t'_1 t} \quad \text{REDUCE_CTX_APP_FUN} \\
\\
\frac{t_1 \longrightarrow t'_1}{v t_1 \longrightarrow v t'_1} \quad \text{REDUCE_CTX_APP_ARG} \\
\\
\frac{t_1 \longrightarrow t'_1}{t_1 [T] \longrightarrow t'_1 [T]} \quad \text{REDUCE_CTX_TYPE_FUN} \\
\\
\frac{t \longrightarrow t'}{\{l_1 = v_1, \dots, l_m = v_m, l = t, l'_1 = t'_1, \dots, l'_n = t'_n\} \longrightarrow \{l_1 = v_1, \dots, l_m = v_m, l = t', l'_1 = t'_1, \dots, l'_n = t'_n\}} \quad \text{REDUCE_C}
\end{array}$$

$$\frac{t_1 \longrightarrow t'_1}{\mathbf{let} \, p = t_1 \, \mathbf{in} \, t_2 \longrightarrow \mathbf{let} \, p = t'_1 \, \mathbf{in} \, t_2} \quad \text{REDUCE_CTX_LET_BINDING}$$

$\mathbf{match} \, (p, v) = \sigma$

$$\frac{}{\mathbf{match} \, (x : T, v) = [x \mapsto v]} \quad \text{M_VAR}$$

$$\frac{\forall i \in 1..m. \exists j \in 1..n. (l_i = k_j \wedge \mathbf{match} \, (p_i, v_j) = \sigma_i)}{\mathbf{match} \, (\{l_1 = p_1, \dots, l_m = p_m\}, \{k_1 = v_1, \dots, k_n = v_n\}) = \sigma_1, \dots, \sigma_m} \quad \text{M_RCD}$$

Definition rules: 48 good 0 bad
Definition rule clauses: 99 good 0 bad