Thor: Wielding Hammers to Integrate Language Models and Automated Theorem Provers

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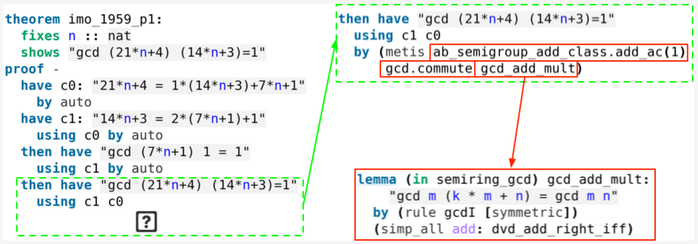
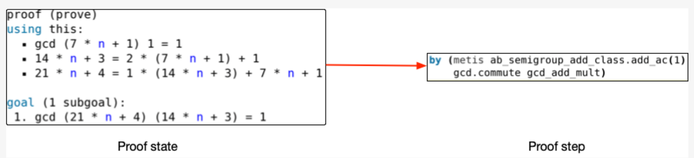
Proof assistant and premise selection

- Proof assistants (e.g, Isabelle) allow mathematical theorems to be proved rigorously.
- We need to select premises (lemmas and definitions) when proving theorems. You don't want to prove every theorem from scratch.



Premise selection is a bottleneck for language models to prove theorems

- Language Models (LMs) predict the next proof step given the proven facts and the remaining objectives. They achieved SoTA in multiple proof assistants [1,2,3].



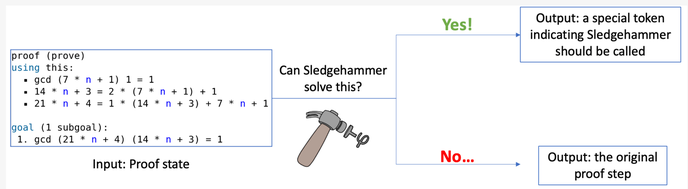
- Premise selection remains a hard retrieval problem for LMs: thousands of lemmas and definitions cannot be crammed into their context windows.
- Empirically, when a LM tries to select premises, 98.1% of the time it produces syntax errors.

Sledgehammer: premise selection based on modern automated theorem provers (ATPs)

- Automated theorem provers can solve SAT, SMT, TPTP, etc. problems that contain millions of variables.
- Proof assistants use Sledgehammer to tackle the premise selection problem by relegating it to ATPs.
- But Sledgehammer is not great with problems involving high-level reasoning, e.g., induction.

Thor: Combining language models and ATPs

- Language models are good at high-level steps (e.g., induction), while ATPs excel at 'low-level' ones (e.g., premise selection).
- Train language model to decide whether to hammer with ATPs.



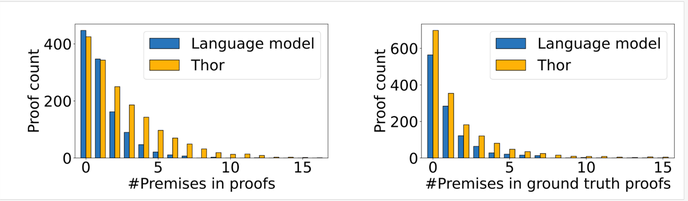
Experiments

- Language model: decoder-only transformer with 700M parameters. Pre-trained on the Github+arXiv subsets of the Pile dataset.
- Sledgehammer: default setup in Isabelle2021.
- Data: Isabelle standard library + the Archive of Formal Proofs (200K theorems, 3M LoC).
- Test set: a suite of 1000 problems from the Archive of Formal Proofs.

Method	Success rate on the test suite (%)
Baselines	
Sledgehammer	25.7
Vanilla language model	39.0
Sledgehammer U Vanilla language model	48.8
Main results	
Thor	57.0

Table 1, Thor solves 8.2% more problems than Sledgehammer and language model naively combined

Is Thor better at premise selection?



Takeaways

- Premise selection is important and difficult. Vanilla pre-trained language models are no good at it.
- Don't throw away the symbolic tools when you have a language model! Integrate them together for better performance.

Let's talk about:

- What's the next step in machine mathematics?
- How could other symbolic tools be integrated with language models?

References

[1] Jiang, Albert Qiaochu, et al. "Language models of Isabelle proofs." *6th Conference on Artificial Intelligence and Theorem Proving*. 2021.

[2] Polu, Stanislas, and Ilya Sutskever. "Generative language modeling for automated theorem proving." *arXiv preprint arXiv:2009.03393* (2020).

[3] Han, Jesse Michael, et al. "Proof Artifact Co-Training for Theorem Proving with Language Models." *International Conference on Learning Representations*. 2021.