MetiTarski: An Automatic Prover for Real-Valued Special Functions

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Many branches of mathematics, engineering and science require reasoning about *special functions:* logarithms, sines, cosines, etc. Few techniques are known for proving statements involving such functions. We have implemented an automatic theorem prover that works by eliminating special functions, substituting rational function upper or lower bounds. It transforms parts of the problem into polynomial inequalities, which it tries to refute by calling QEPCAD, a decision procedure for the theory of *real closed fields* (RCF). MetiTarski delivers machine-readable proofs.

MetiTarski is based on Metis, a resolution theorem prover developed by Joe Hurd. Our approach requires a full first-order theorem prover even to prove simple inequalities. The bounds typically have side conditions that must be proved. Case analysis is necessary when eliminating division and often when substituting bounds, for example when combining intervals.

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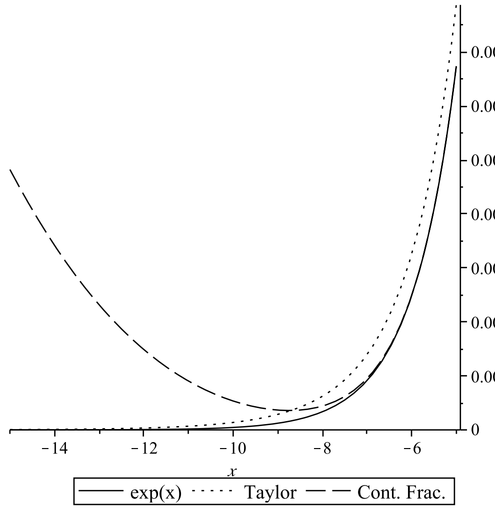
The diagram illustrates some complications that arise when using algebraic approximations to special functions. It shows two upper bounds for the exponential function: one based on a Taylor series and the other on a continued fraction. Both bands are extremely accurate over certain intervals but elsewhere can veer wildly astray. A typical proof will use a combination of bounds in order to get a good fit everywhere.



We have collected over 800 problems, some from mathematical reference works and others from hybrid and control systems. MetiTarski can prove the problems shown below in seconds.

$$0 < t \wedge 0 < v_f \Longrightarrow ((1.565 + .313 v_f)\cos(1.16t) + (.01340 + .00268 v_f)\sin(1.16t))e^{-1.34t} - (6.55 + 1.31 v_f)e^{-.318t} + v_f + 10 \geq 0$$

$$0 \leq x \wedge 0 \leq y \Longrightarrow y \tanh(x) \leq \sinh(yx)$$

$$0 \leq x \wedge x \leq 289 \wedge s^2 + c^2 = 1 \Longrightarrow 1.51 - .023e^{-.019x} - (2.35c + .42s)e^{.00024x} > -2$$

$$0 \leq x \wedge x \leq 1.46 \times 10^{-6} \Longrightarrow (64.42\sin(1.71 \times 10^6 x) - 21.08\cos(1.71 \times 10^6 x))e^{9.05 \times 10^5 x} + 24.24e^{-1.86 \times 10^6 x} > 0$$

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