

Binomial distribution (solid curve in histogram for 0 rotations):

$$
\begin{equation*}
f(x)=\frac{N!}{m!(N-m)!} p^{m}(1-p)^{(N-m)} \tag{1}
\end{equation*}
$$

where $x=m / N$ (e.g. the fraction of 'heads' outcomes in $N$ coin tosses).

Logic for computing raw Hamming Distance scores, incorporating masks:

$$
\begin{equation*}
H D_{\mathrm{raw}}=\frac{\|(\operatorname{code} A \otimes \operatorname{code} B) \cap \operatorname{mask} A \cap \operatorname{mask} B\|}{\|\operatorname{mask} A \cap \operatorname{mask} B\|} \tag{2}
\end{equation*}
$$

where $\otimes$ is Exclusive-OR, $\cap$ is AND, and $\|\quad\|$ is the count of 'set' bits.

Score re-normalisation to compensate for number of bits compared:

$$
\begin{equation*}
H D_{\text {norm }}=0.5-\left(0.5-H D_{\text {raw }}\right) \sqrt{\frac{n}{911}} \tag{3}
\end{equation*}
$$

Raw Binomial ( $\mathrm{p}=0.5, \mathrm{~N}=249 \mathrm{DoF}$ ) and its Extreme-value Distribution


200 Billion Iris Cross-Comparisons, 7 Rotations, UAE Database


The new distribution after $k$ rotations of IrisCodes in the search process still has a simple analytic form that can be derived theoretically. Let $f_{0}(x)$ be the raw density distribution obtained for the $H D_{\text {norm }}$ scores between different irises after comparing them only in a single relative orientation; for example, $f_{0}(x)$ might be the binomial defined in Eqn (1). Then $F_{0}(x)$, the cumulative of $f_{0}(x)$ from 0 to $x$, becomes the probability of getting a False Match in such a test when using $H D_{\text {norm }}$ acceptance criterion $x$ :

$$
\begin{equation*}
F_{0}(x)=\int_{0}^{x} f_{0}(x) d x \tag{4}
\end{equation*}
$$

or, equivalently,

$$
\begin{equation*}
f_{0}(x)=\frac{d}{d x} F_{0}(x) \tag{5}
\end{equation*}
$$

Clearly, then, the probability of not making a False Match when using decision criterion $x$ is $1-F_{0}(x)$ after a single test, and it is $\left[1-F_{0}(x)\right]^{k}$ after carrying out $k$ such tests independently at $k$ different relative orientations. It follows that the probability of a False Match after a "best of $k$ " test of agreement, when using $H D_{\text {norm }}$ criterion $x$, regardless of the actual form of the raw unrotated distribution $f_{0}(x)$, is:

$$
\begin{equation*}
F_{k}(x)=1-\left[1-F_{0}(x)\right]^{k} \tag{6}
\end{equation*}
$$

and the expected density $f_{k}(x)$ associated with this cumulative is:

$$
\begin{align*}
f_{k}(x) & =\frac{d}{d x} F_{k}(x) \\
& =k f_{0}(x)\left[1-F_{0}(x)\right]^{k-1} \tag{7}
\end{align*}
$$

# Observed False Match Rates in 200 billion comparisons 

| HD Criterion Policy | Observed False Match Rate |
| :---: | :---: |
| 0.220 | $0 \quad$ (theor: 1 in $5 \times 10^{15}$ ) |
| 0.225 | $0 \quad$ (theor: 1 in $\left.1 \times 10^{15}\right)$ |
| 0.230 | 0 |
| 0.235 | 0 |
| (theor: 1 (theor: 1 in $3 \times 10^{14} 9 \times 10^{13}$ ) |  |
| 0.240 | 0 |
| (theor: 1 in $3 \times 10^{13}$ ) |  |
| 0.245 | 0 |
| (theor: 1 in $\left.8 \times 10^{12}\right)$ |  |
| 0.250 | 0 |
| (theor: 1 in $2 \times 10^{12}$ ) |  |
| 0.255 | 0 |
| (theor: 1 in $\left.7 \times 10^{11}\right)$ |  |
| 0.262 | 1 in 200 billion |
| 0.267 | 1 in 50 billion |
| 0.272 | 1 in 13 billion |
| 0.277 | 1 in 2.7 billion |
| 0.282 | 1 in 284 million |
| 0.287 | 1 in 96 million |
| 0.292 | 1 in 40 million |
| 0.297 | 1 in 18 million |
| 0.302 | 1 in 8 million |
| 0.307 | 1 in 4 million |
| 0.312 | 1 in 2 million |
| 0.317 | 1 in 1 million |

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