



**Binomial distribution** (solid curve in histogram for 0 rotations):

$$f(x) = \frac{N!}{m!(N-m)!} p^m (1-p)^{(N-m)}$$
(1)

where x = m/N (e.g. the fraction of 'heads' outcomes in N coin tosses).

Logic for computing raw Hamming Distance scores, incorporating masks:

$$HD_{\rm raw} = \frac{\|(codeA \otimes codeB) \cap maskA \cap maskB\|}{\|maskA \cap maskB\|}$$
(2)

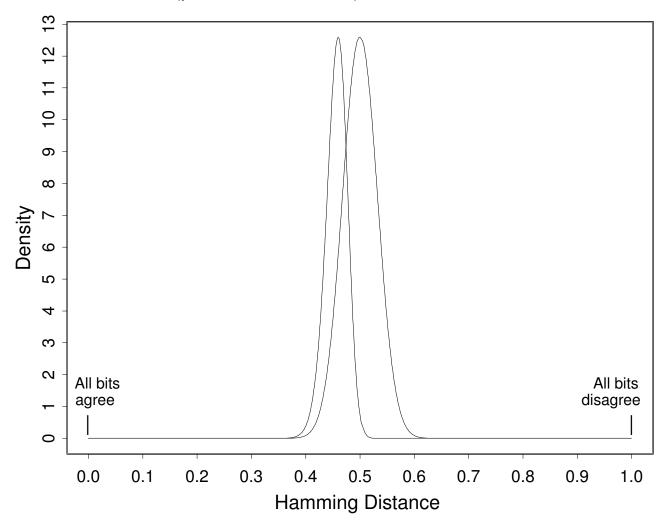
where  $\otimes$  is Exclusive-OR,  $\cap$  is AND, and  $\parallel \parallel$  is the count of 'set' bits.

Score re-normalisation to compensate for number of bits compared:

$$HD_{\rm norm} = 0.5 - (0.5 - HD_{\rm raw})\sqrt{\frac{n}{911}}$$
 (3)

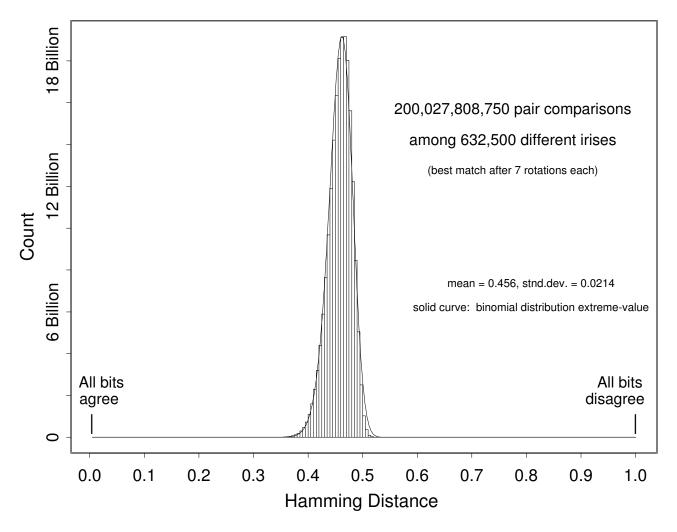


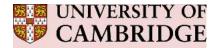
Raw Binomial (p=0.5, N=249 DoF) and its Extreme-value Distribution





## 200 Billion Iris Cross-Comparisons, 7 Rotations, UAE Database





The new distribution after k rotations of IrisCodes in the search process still has a simple analytic form that can be derived theoretically. Let  $f_0(x)$  be the raw density distribution obtained for the  $HD_{norm}$  scores between different irises after comparing them only in a single relative orientation; for example,  $f_0(x)$  might be the binomial defined in Eqn (1). Then  $F_0(x)$ , the cumulative of  $f_0(x)$  from 0 to x, becomes the probability of getting a False Match in such a test when using  $HD_{norm}$  acceptance criterion x:

$$F_0(x) = \int_0^x f_0(x) dx$$
 (4)

or, equivalently,

$$f_0(x) = \frac{d}{dx} F_0(x) \tag{5}$$

Clearly, then, the probability of *not* making a False Match when using decision criterion x is  $1 - F_0(x)$  after a single test, and it is  $[1 - F_0(x)]^k$  after carrying out k such tests independently at k different relative orientations. It follows that the probability of a False Match after a "best of k" test of agreement, when using  $HD_{norm}$  criterion x, regardless of the actual form of the raw unrotated distribution  $f_0(x)$ , is:

$$F_k(x) = 1 - [1 - F_0(x)]^k$$
(6)

and the expected density  $f_k(x)$  associated with this cumulative is:

$$f_k(x) = \frac{d}{dx} F_k(x) = k f_0(x) [1 - F_0(x)]^{k-1}$$
(7)



## Observed False Match Rates in 200 billion comparisons

HD Criterion Policy	Observed False Match Rate
0.220	0 (theor: 1 in $5 \times 10^{15}$ )
0.225	0 (theor: 1 in $1 \times 10^{15}$ )
0.230	0 (theor: 1 in $3 \times 10^{14}$ )
0.235	0 (theor: 1 in $9 \times 10^{13}$ )
0.240	0 (theor: 1 in $3 \times 10^{13}$ )
0.245	0 (theor: 1 in $8 \times 10^{12}$ )
0.250	0 (theor: 1 in $2 \times 10^{12}$ )
0.255	0 (theor: 1 in $7 \times 10^{11}$ )
0.262	1 in 200 billion
0.267	1 in 50 billion
0.272	1 in 13 billion
0.277	1 in 2.7 billion
0.282	1  in  284  million
0.287	1 in 96 million
0.292	1 in 40 million
0.297	1 in 18 million
0.302	1 in 8 million
0.307	1 in 4 million
0.312	1 in 2 million
0.317	1 in 1 million



## References

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