Operational Versus Denotational Methods in the Semantics of Higher Order Languages

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Abstract. In the last few years increasing use has been made of structural operational semantics to study aspects of programming languages which traditionally have been analysed via denotational semantics. (The articles in the recent collection by Gordon and Pitts 1998 are a good illustration of this development and its applications.) Since there are more or less adequate denotational models for programming language features such as higher order functions and procedures, recursive definitions, local state, and data abstraction, one might wonder why one should consider syntactic methods at all. This tutorial talk will attempt to explain why one should.

> *'But once feasibility has been checked by an operational model, operational reasoning should be immediately abandoned; it is essential that all subsequent reasoning, calculation and design should be conducted in each case at the highest possible level of abstraction.'* Hoare (1996, page 182)

The advice in the second half of the above quotation—that one should always strive for the highest possible level of abstraction—is very sound. Nevertheless, I am going to try to persuade you to ignore the first half and embrace, rather than abandon, operational reasoning. The justification for this lies in the claim that operational semantics can (sometimes—we are still learning how) be presented at a sufficient 'level of abstraction' to support quite palatable methods of reasoning, calculation and design. I plan to cover some of the following topics.

- Structural operational semantics—a brief survey, including the use of *structural congruence relations*, arising from work in concurrency theory (Berry and Boudol 1992, Milner 1990), to simplify the auxiliary syntactic structures (environments, memories, program stacks, multisets of program threads, etc) needed in a semantic specification.
- The use of *evaluation contexts* to aid inductive reasoning about transition relations.(Felleisen and Hieb 1992,Wright and Felleisen 1994, Harper and Stone 1996.)
- Operational analogues of the domain-theoretic concepts of *least fixed points, continuity,* and the *minimal invariant* property of recursively defined domains. (Mason, Smith, and Talcott 1996, Smith 1998.)
- Applications of operationally-based *logical relations.* (Pitts 1997, Birkedal and Harper 1997, Pitts and Stark 1998, Pitts 1998.)

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