▸ ANUJ DAWAR, *The power of fixed-point logic with counting.*
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Fixed-point Logic with Counting (FPC) is the extension of first-order logic with (1) the ability to define relations inductively and (2) the ability to count. The latter is in the form of operators that construct terms representing the finite cardinalities of definable sets. FPC has been much studied in finite model theory since it was first proposed by Neil Immerman as a logic that might express all the polynomial-time properties of finite unordered structures (see, for instance, [1]). It was eventually proved (by Cai, Frer and Immerman [6]) that there are polynomial-time graph properties that are not expressible in FPC. A number of other results establishing limits on the expressive power of FPC have since been proved [7, 4]. Nonetheless, FPC is a powerful and natural fragment of the complexity class P and a number of recent results demonstrate this. In particular, we consider the following.

1. Grohe [8] establishes that on any class of finite graphs that is closed under the graph minor relation (and that is not the class of all graphs), all polynomial-time decidable properties are definable in FPC. In particular, any class of graphs closed under the graph minor relation is itself definable in FPC. This result builds on Robertson and Seymour's graph structure theory and constructs a *definable* version of their key structure theorem.
2. Anderson et al. [3] show that FPC can define the feasiblity of linear programs. More generally, linear optimization problems can be expressed in FPC as long as a *separation oracle* for the problem is definable. This is used to establish that the graph perfect matching problem is expressible in FPC, settling a long-standing open problem due to Blass, Gurevich and Shelah [5].
3. Anderson and Dawar [2] establish that the expressive power of FPC can be exactly characterised by a natural circuit complexity class based on a *symmetry* restriction. This shows that inexpressibility results established in finite model theory can be interpreted as circuit lower bounds.

The talk reviews the history and background of fixed-point logic with counting and then give a brief account of the three results above.

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