Computations on topological algebras

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Abstract

Computations on topological algebras cannot be performed in general since the spaces may be uncountable. By considering computations on approximations rather than on the actual elements it is possible to establish a theory for computations on topological algebras. One way to do this is if the approximations constitute the compact elements of a Scott–Ershov domain, we call this a *domain representation* of the space. The natural computability theory of domains may be imported into the topological spaces.

Introduction of computability to topological algebras via domain representations can be shown to work for a large class of metric spaces, the *effective metric spaces*, including several L^p-spaces. In fact, domain representations can be given for any T_0 topological space. However, the domains constructed for general topological spaces are not always countably based and are therefore unsuitable for computations.

The general computability introduced by a domain representation of the reals correspond to the classical theory of Computable Analysis. However, there exists many sources for uncountable topological algebras, e.g., streams. We view streams as arbitrary functions from time to data. Streams may model both continuous and discrete phenomena with respect to both time and data. Hence, discontinuous streams (functions) appear naturally, for example, digital signals seen over continuous time constitute streams that are discontinuous unless they are constant. By introducing a notion of streams being *approximatively represented* we may actually consider computations containing discontinuous streams (functions).

The talk will give a survey of computations on topological algebras in general and in particular computability by domain representations. Some case studies as, for example, streams and some applications will also be presented.