COMPUTER SCIENCE TRIPOS Part II – 2023 – Paper 9

13 Types (nk480)

(a) Derive the following entailments with the natural deduction system for classical logic.

  (i) Show $\neg(A \vee B); \cdot \vdash \neg A$ true. [5 marks]

  (ii) Show $\cdot; \neg A \vee \neg B \vdash A$ true. [5 marks]

(b) (i) Using $fold : \forall a.\, a \to (X \to a \to a) \to \mathsf{List}_X \to a$,
     $cons : X \to \mathsf{List}_X \to \mathsf{List}_X$ and $nil : \mathsf{List}_X$, write a System F function which appends two lists. [1 mark]

  (ii) Give an OCaml data structure corresponding to the following Church encoding:
$$\forall a.\, a \to (a \to X \to a \to a) \to a$$
  [2 marks]

  (iii) Give a System F term which converts an element t of the type in part (ii) of this question into a list with the same elements. [3 marks]

(c) Consider the following two Agda proofs:

$$
\begin{array}{llll}
\mathsf{unitl} & : & \forall x \to 0 + x \equiv x & \\
\mathsf{unitl}\ x & = & \mathsf{refl}(x) &
\end{array}
\qquad
\begin{array}{lll}
\mathsf{unitr} & : & \forall x \to x + 0 \equiv x \\
\mathsf{unitr}\ 0 & = & \mathsf{refl}(0) \\
\mathsf{unitr}\ (\mathsf{s}\ n) & = & \mathsf{cong\ s}\ (\mathsf{unitr}\ n)
\end{array}
$$

Explain why they are different. [4 marks]