

4 Denotational Semantics (mpf23)

Define Σ to be the flat domain $\{\top\}_\perp$.

(a) Let P be a poset with partial order \sqsubseteq and let S be a subset of P .

Define $\mathcal{M}(S)$ to be the property of S given by $\forall x \in S. \forall y \in P. x \sqsubseteq y \Rightarrow y \in S$.

Prove that $\mathcal{M}(S)$ holds if, and only if, there exists a monotone function $f : P \rightarrow \Sigma$ such that $f^{-1}\{\top\} = S$. [8 marks]

(b) Let D be a cpo.

(i) For subsets S of D , define a property $\mathcal{C}(S)$ in terms of the cpo structure of D such that

(†) $\left\{ \begin{array}{l} \mathcal{C}(S) \text{ holds if, and only if, there exists a continuous function} \\ f : D \rightarrow \Sigma \text{ such that } f^{-1}\{\top\} = S. \end{array} \right.$ [4 marks]

(ii) Prove (†) above. [8 marks]