## COMPUTER SCIENCE TRIPOS Part IA – 2023 – Paper 2

## 9 Discrete Mathematics (mpf23)

(a) Let  $B \subseteq \{<,>\}^*$  be the set inductively defined by the axiom and rule below

and let  $f: B \to B$  be the inductively defined function given by

$$f(<>) = <>$$
 ,  $f(< l r >) = < f(r) f(l) >$ 

- (i) State whether or not f is the identity function on B, and prove your claim. [2 marks]
- (*ii*) State whether or not f is a bijection, and prove your claim. [5 marks]
- (b) Let  $L \subseteq \{a\}^* \times \mathbb{N}$  be the relation inductively defined by the axiom and rule below (u, m) = (v, n)

$$(u, m) \quad (v, n) \quad (uv, m+n)$$

- (i) Give a pair in  $\{a\}^* \times \mathbb{N}$  together with two different derivations that show that the pair is in L. [2 marks]
- (*ii*) Prove that, for all  $(w, k) \in L, k \ge 1$ . [5 marks]
- (*iii*) Prove that, for all  $n \in \mathbb{N}$ ,  $(\varepsilon, n) \notin L$ . [6 marks]

[*Hint*: Argue by contradiction.]