8 Semantics of Programming Languages (nk480)

Consider the following grammar of types and subtyping relation:

$$A, B \quad ::= \quad \mathsf{int} \ \mid \ \{l_1 : A_1, \ldots, l_n : A_n\} \ \mid \ A \to B$$

$$\frac{}{\mathsf{int} \leq \mathsf{int}}\leq\mathsf{int}$$

$$\frac{B_1 \leq A_1 \qquad A_2 \leq B_2}{A_1 \to A_2 \leq B_1 \to B_2}\leq\to$$

$$\frac{n \leq m \qquad A_1 \leq B_1 \qquad \ldots \qquad A_n \leq B_n}{\{l_1 : A_1, \ldots, l_n : A_n\} \leq \{l_1 : B_1, \ldots, l_m : B_m\}}\leq\mathsf{rec}$$

We did not include explicit rules for reflexivity or transitivity in our subtyping relation. However, we can prove that they are *admissible* – that is, the subtyping judgement already has enough power to simulate them.

In the following questions, you may make use of any needed inversion properties without giving explicit proofs for them.

(*a*) Prove that for each type A, a subtyping derivation $A \leq A$ can be constructed. [5 marks]

(*b*) Prove that for all A, B, C, if $A \leq B$ and $B \leq C$ are derivable, then $A \leq C$ is derivable. [15 marks]