# CST0
# COMPUTER SCIENCE TRIPOS Part IA

Tuesday 7 June 2022 14:00 to 17:00 BST

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit each question answer in a **separate** PDF. As the file name, use your candidate number, paper and question number (e.g., `1234A-p2-q6.pdf`). Also write your candidate number, paper and question number at the start of each PDF.*

**You must follow the official form and conduct instructions for this online examination**

## SECTION A

### 1 Digital Electronics

($a$) Show using Boolean algebra

($i$) $\overline{X}.Y \oplus X.\overline{Y} = \overline{X}.Y + X.\overline{Y}$

($ii$) $\overline{X}.\overline{Y} \oplus X.Y = \overline{X}.\overline{Y} + X.Y$

[4 marks]

($b$) Using the results in Part ($a$) or otherwise, express the four-variable function

$$F(A, B, C, D) = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{C}.\overline{D} + \overline{A}.B.C.D + A.B.\overline{C}.D + A.C.\overline{D} + A.\overline{B}.C$$

as the Exclusive OR of three 2-variable AND terms, i.e., $F = X_1.X_2 \oplus X_3.X_4 \oplus X_5.X_6$ where each $X_i$ is either one of the four variables or its complement.

[4 marks]

($c$) ($i$) Simplify the four-variable function

$$G(A, B, C, D) = \sum(0, 2, 6, 7, 8, 9, 10, 13, 15)$$

using the Quine-McCluskey (Q-M) method. The numbers in the summation are the decimal representations of the minterms of $G$ (where $A$ represents the most-significant bit of the equivalent binary representation).

[9 marks]

($ii$) How many equal-complexity solutions exist in total? Justify your answer.

[3 marks]

## 2 Digital Electronics

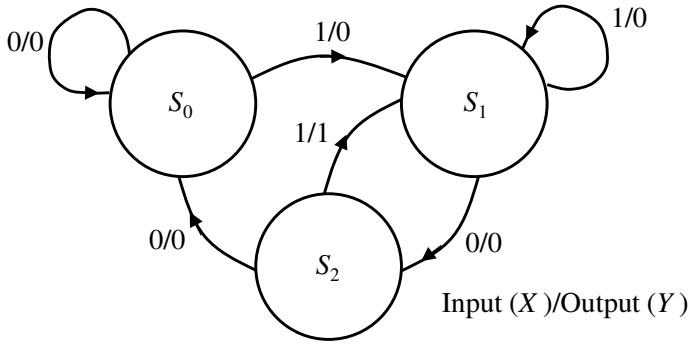
($a$) A (fictional) edge-triggered $UV$ flip-flop has inputs $U$ and $V$ and output $Q$. Its state-transition table is given by:

| Current state ($Q$) | Next state ($Q'$) | | | |
|---|---|---|---|---|
| $UV =$ | 00 | 01 | 10 | 11 |
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |

($i$) Draw the state-transition diagram for the $Q$ output. [3 marks]

($ii$) For an implementation based on a D-type flip-flop, determine the simplified Boolean equation in sum-of-products form for the next-state ($Q'$) logic. [2 marks]

($b$) Consider the following state machine:




($i$) Assuming that the machine starts in state $S_0$ and that the input data sequence at input ($X$) is appropriately synchronised with the state machine clock, determine the next-state and output sequences for the input sequence 0101011011011. What operation does the machine perform? [5 marks]

($ii$) For an implementation based on two D-type flip-flops (labelled $A$ and $B$), determine simplified Boolean expressions for the next-state and output combinational logic, assuming the state assignment $S_0 = 00$, $S_1 = 01$ and $S_2 = 10$ is used, where a state is labelled $Q_AQ_B$ in terms of the flip-flop outputs. [4 marks]

($iii$) For an alternative one-hot implementation based on D-type flip-flops, determine expressions for the next-state and output logic. [4 marks]

($iv$) What feature, inherent in the proposed state-machine design, may give rise to problems at the output $Y$? How might this be addressed? [2 marks]

## SECTION B

### 3 Operating Systems

(*a*) Two common ways of providing memory protection between processes are segmentation and paging. Modern systems would typically implement software segments using paging. What resource does this waste compared to using pure segments? What cost does alleviating this wastage impose? [2 marks]

(*b*) A 32-bit system with 32-bit virtual addressing uses 4 kB pages to match 4 kB frames.

(*i*) Explain why a thoughtful engineer might choose 4-byte page table entries (PTE) and use a two-level page table in this system. [6 marks]

(*ii*) A radical engineer decides they want finer-grained protection of memory and proposes reducing the page size to 1 kB. State what challenge this introduces, suggest two different ways it might be addressed, and discuss the associated costs. [7 marks]

(*c*) An intrepid software engineer discovers some old hardware with support for segmentation, and decides to implement software paging using segments. Propose how they might do this, and discuss how your proposal would perform, and what factors would affect its performance. [5 marks]

## 4 Operating Systems

(*a*) On creation, a UNIX process normally inherits three file descriptors from its parent: *stdin*, *stdout*, and *stderr*.

(*i*) Which additional kernel interactions are involved if the user redirects a process' *stdout* to a file? [4 marks]

(*ii*) Why is it typically more efficient to connect two processes using a pipe rather than redirecting the first's *stdout* to a file to which the second's *stdin* is also redirected? [2 marks]

(*b*) UNIX file descriptors are sometimes described as *capabilities*.

(*i*) In what ways do they provide capability functionality? [4 marks]

(*ii*) In what ways do they differ from capabilities? [4 marks]

(*c*) A common approach to handling the situation where there is no process either scheduled or waiting to be scheduled is to introduce an *Idle* process. An intrepid developer decides that when the Idle process is scheduled it could usefully carry out *filesystem defragmentation*, by rearranging files on disk to ensure each is stored in contiguous blocks. Explain three disadvantages of this use of the Idle process in a modern computer. [6 marks]

## SECTION C

### 5 Software and Security Engineering

(*a*) Describe briefly one positive and one negative effect that social networks and other modern communication platforms may have on the personal safety of their users. [4 marks]

(*b*) Suppose social networks are partly replaced by virtual reality, in which users have binocular headsets that provide the illusion of being immersed in a different 3D universe. Do any arguments from psychology suggest that the positive and negative effects from your answer to Part (*a*) are likely to increase or decrease as a result? [6 marks]

(*c*) You have joined a startup whose goal is to move beyond virtual reality to augmented reality, and supply software for augmented-reality spectacles that overlays text and graphics on the real world as the user views it. Your responsibility is product safety. Describe the safety policy you will set and the software development methodology you will adopt to ensure that the product delivers it as far as reasonably practical. [10 marks]

### 6 Software and Security Engineering

(*a*) Describe how bookkeeping systems protect firms from dishonest staff. [5 marks]

(*b*) Describe, with reference to an example, how such systems may fail to protect staff from a dishonest firm. [5 marks]

(*c*) Do firms that supply or that use bookkeeping systems have adequate incentives to provide enough evidence to all stakeholders? [5 marks]

(*d*) What reasonably practical measures might be put in place to prevent a recurrence of the Post Office Horizon scandal? [5 marks]

## SECTION D

### 7 Discrete Mathematics

($a$) Fix positive integers $m$ and $n$.

($i$) For $k \in \mathbb{Z}$, define $[k]_m$ to be the unique element of $\mathbb{Z}_m$ congruent to $k$ modulo $m$.

Prove that: $\forall\, k, \ell \in \mathbb{Z}.\, [k]_m = [\ell]_m \Leftrightarrow k \equiv \ell \pmod{m}$. [3 marks]

($ii$) Let $f : \mathbb{Z}_m \to \mathbb{Z}_m$ be the function defined by $f(k) = [nk]_m$ and let $+_m : \mathbb{Z}_m \times \mathbb{Z}_m \to \mathbb{Z}_m$ be the function defined by $k +_m \ell = [k + \ell]_m$.

(A) Prove that: $\forall\, k, \ell \in \mathbb{Z}_m.\, f(k +_m \ell) = f(k) +_m f(\ell)$. [3 marks]

(B) Prove that $f$ is a bijection if, and only if, $\gcd(m, n) = 1$. [6 marks]

($b$) Recall that $\mathrm{Bij}(X, Y)$ denotes the set of bijections from a set $X$ to a set $Y$ and that, for $n \in \mathbb{N}$, the set $[n]$ is defined as $\{i \in \mathbb{N} \mid i < n\}$.

($i$) Given a set $A$ such that $0 \notin A$, describe without proof a bijection

$$\mathrm{Bij}\big(\{0\} \cup A\,,\, \{0\} \cup A\big) \to (\{0\} \cup A) \times \mathrm{Bij}(A, A)$$

[*Hint:* For $f \in \mathrm{Bij}\big(\{0\} \cup A\,,\, \{0\} \cup A\big)$ consider both $f(0)$ and $f^{-1}(0)$.] [4 marks]

($ii$) Using the above or otherwise, prove that: $\forall\, n \in \mathbb{N}.\, \mathrm{Bij}\big([n]\,,\, [n]\big) \cong [n!]$. [4 marks]

## 8 Discrete Mathematics

(*a*) Without using the Fundamental Theorem of Arithmetic, prove that for all positive integers $a, b, c$, if $\gcd(b, c) = 1$ then $\gcd(a, b \cdot c) = \gcd(a, b) \cdot \gcd(a, c)$. [4 marks]

(*b*) Let $k$ be a fixed integer.

Set $p_0 = q_0 = 1$ and, for $n \in \mathbb{N}$, let $p_{n+1} = p_n + k\, q_n$ and $q_{n+1} = p_n + q_n$.

For $n \in \mathbb{N}$, define

$$r_n = \mid k\,(q_n)^2 - (p_n)^2 \mid$$

(*i*) For $n \in \mathbb{N}$, give a closed-form expression $s_n$ defined in terms of $k$ and $n$ such that $s_n = r_n$. [3 marks]

(*ii*) Prove that $s_n = r_n$ for all $n \in \mathbb{N}$. [5 marks]

(*c*) Fix sets $A$ and $B$.

Consider a set $P$ together with functions $p : P \to A$ and $q : P \to B$ such that for all sets $X$ and for all functions $f : X \to A$ and $g : X \to B$ there exists a unique function $u\langle f, g\rangle : X \to P$ satisfying $p \circ u\langle f, g\rangle = f$ and $q \circ u\langle f, g\rangle = g$.

(*i*) Define a function from $P$ to the product $A \times B$. [1 mark]

(*ii*) Define a function from the product $A \times B$ to $P$. [1 mark]

(*iii*) Prove that $u\langle p, q\rangle : P \to P$ is the identity function. [2 marks]

(*iv*) Prove that $P$ and the product $A \times B$ are isomorphic. [4 marks]

## 9 Discrete Mathematics

($a$) For sets $A$ and $B$, recall that $A \Rightarrow B$ denotes the set of all functions from $A$ to $B$ and that $f : A \to B$ states that $f$ is a function from $A$ to $B$.

($i$) Let $R$ be a set.

For a set $X$ define $\eta_X : X \to \big((X \Rightarrow R) \Rightarrow R\big)$ by

$$\eta_X(x)(f) \;=\; f(x)$$

and define $F : \Big(\big((X \Rightarrow R) \Rightarrow R\big) \Rightarrow R\Big) \to (X \Rightarrow R)$ by

$$F(\varphi)(x) \;=\; \varphi\big(\eta_X(x)\big)$$

Prove that $F$ is surjective. [*Hint:* $F$ is actually a retraction.] [6 marks]

($ii$) Using the above, or otherwise, prove that for all sets $X$ and $R$, if there is a surjection from $X$ to $\Big(\big((X \Rightarrow R) \Rightarrow R\big) \Rightarrow R\Big)$ then $R$ is a singleton. You may use standard results provided that you state them clearly. [4 marks]

($b$) For sets $\Sigma$ and $A$, let $a \in A$ and $f : \Sigma \times \Sigma^* \times A \to A$. Let $R$ be the subset of $\Sigma^* \times A$ inductively defined by the axiom

$$\frac{}{(\,\varepsilon\,,\,a\,)}$$

and the rule

$$\frac{(\,w\,,\,x\,)}{\big(\,sw\,,\,f(s,w,x)\,\big)} \qquad (s \in \Sigma, w \in \Sigma^*, x \in A)$$

Prove that:

($i$) $R$ is total; that is, $\forall\, w \in \Sigma^*.\ \exists\, x \in A.\ (w,x) \in R$. [4 marks]

($ii$) $R$ is functional; that is, $\forall\, (w,x) \in R.\ \forall\, y \in A.\ (w,y) \in R \Rightarrow y = x$. [6 marks]

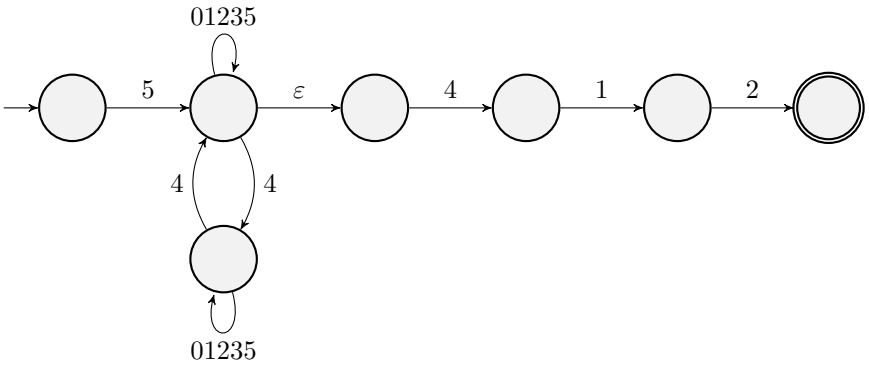
## 10 Discrete Mathematics

All the formal languages associated with finite automata in this question are defined over the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5\}$. [*Note:* Ensure that any answer DFA you provide is actually a DFA.]

($a$) Build a 4-state DFA $A_0$ to recognise the set of strings that start with 5 and have an even number of 0s. [2 marks]

($b$) Build a 6-state DFA $A_1$ to recognise the same language as $A_0$. Every state in $A_1$ must be reachable by some string in $\Sigma^*$. [5 marks]

($c$) Here is a 7-state NFA$^\varepsilon$ $A_2$.




($i$) Find strings $x, y, z \in \Sigma^*$ such that the following statement is true: "$A_2$ accepts all and only the strings that start with $x$, contain an odd number of $y$ and end with $z$". [2 marks]

($ii$) Build a DFA $A_3$ with no more than seven states that recognises the same language as $A_2$. [*Hint:* Check whether the property in part ($c$)($i$) still holds for your $A_3$.] [7 marks]

($d$) For each of the following four strings, state which of $A_0$ and $A_2$ recognise it. [*Note:* Spaces have been inserted for legibility but have no other significance.]

($i$) 5234 5543 2100 1412

($ii$) 5555 5500 5031 0041 2

($iii$) 5430 4041 2

($iv$) 5421

[4 marks]

**END OF PAPER**