COMPUTER SCIENCE TRIPOS Part II – 2021 – Paper 9

14 Quantum Computing (sjh227)

(a) Find the eigenvectors, eigenvalues and spectral decomposition of the observable

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and give the outcome of measuring the expectation of the observable on the states:

(i) $|0\rangle$ (ii) $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ (iii) $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

[8 marks]

(b) A quantum mechanical system has Hamiltonian

$$\mathbf{H} = \mathbf{H}_1 + 2\mathbf{H}_2$$

It is desired to use a quantum computer to approximately simulate the operator e^{-iHt} for some t. It is possible to build quantum circuits U_1 and U_2 to perform the operations

$$U_1 = e^{-i\mathbf{H}_1 t}$$
$$U_2 = e^{-i\mathbf{H}_2 t}$$

Give a circuit, U, consisting of one of more instances of U_1 and U_2 that approximates e^{-iHt} such that $e^{-iHt} - U = \mathcal{O}(t^3)$. Show your calculations to verify that the circuit does indeed achieve this. [8 marks]

(c) Quantum Phase Estimation can be used to estimate the ground state energy of quantum mechanical systems. The Inverse Quantum Fourier Transform is a key component of Quantum Phase Estimation. Give the circuit for the 2-qubit Inverse Quantum Fourier Transform using only gates from the set $\{H, CT, CNOT\}$, where CT is a controlled T gate. [4 marks]