15 Types (nk480)

(*a*) In the calculus of proofs and refutations, suppose that $\Gamma; \Delta \vdash A$ true and $\Gamma, A; \Delta \vdash C$ true. Show that $\Gamma; \Delta \vdash C$ true is derivable. [*Hint:* Recall that weakening is admissible in this calculus.] [8 marks]

(*b*) In System F, consider an arbitrary type A.

(*i*) Give two terms $f : A \to \forall a.\, (A \to a) \to a$ and $g : (\forall a.\, (A \to a) \to a) \to A$. [3 marks]

(*ii*) Carefully explain what this tells you about the relationship between the types A and $\forall a.\, (A \to a) \to a$. [4 marks]

(*c*) Consider the following piece of Agda code, where `Nat` is the type of natural numbers:

```
X : (P : Nat → Set) →
    P 0 →
    ((n : Nat) → P n → P (1 + n)) →
    (k : Nat) → P k
X P base step zero = base
X P base step (suc n) = step n (X P base step n)
```

(*i*) Explain what the `X` function means in logical terms. [2 marks]

(*ii*) Explain what the `X` function does in terms of functional programming. [3 marks]