COMPUTER SCIENCE TRIPOS Part II – 2021 – Paper 8

12 Optimising Compilers (tmj32)

We wish to use abstract interpretation to analyse the construction, modification and traversal of directed graphs. Graph nodes are represented by the following C-like structure; the root of a graph is a pointer to a graph_node:

typedef struct graph_node {
 int value;
 struct graph_node *children;
} graph_node;

Graph nodes are assumed to have a maximum of two children. Exceptions (for example, caused by trying to add a third child to any node, or a search failing to find a node) cause control to transfer out of the program, and do not need to be considered further in the analysis.

- (a) The first analysis consists of identifying whether a graph is actually a tree.
 - (i) Create a three-value abstraction for this analysis, describing abstract values and the concrete values that they represent, and why it is safe. [4 marks]
 - (*ii*) Define the abstract interpretation of the following concrete functions giving a brief explanation for each.
 - (A) Function $create_child(g)$ creates a new graph_node and makes it a child of g, returning the new child. [2 marks]
 - (B) Function $add_child(g,c)$ makes an existing node, c, a child of g, returning g. [3 marks]
 - (C) Function $remove_child(g, c)$ removes node c from g's children, returning g. [2 marks]
 - (D) Function dfs(g, v) locates and returns the first node in a depth-first search starting at g that contains the value v. [2 marks]
- (b) The second analysis consists of calculating the length of the shortest path from a node to any leaf node. Create an abstraction for this analysis and define the abstract interpretation for the four functions in Part (a)(ii). [Hint: consider using a tuple for your abstract values.] [7 marks]