

10 Logic and Proof (lp15)

- (a) A mysterious propositional connective,  $\odot$ , has the following sequent calculus rule,  $(\odot l)$ :

$$\frac{A, B, \Gamma \Rightarrow \Delta \quad \Gamma \Rightarrow \Delta, A, B}{A \odot B, \Gamma \Rightarrow \Delta}$$

Present the corresponding right-side sequent calculus rule,  $(\odot r)$ , along with the truth table for  $\odot$ . [5 marks]

- (b) For the following formula, either exhibit a formal proof (using the sequent calculus, augmented with the  $(\odot l)$  rule above) or exhibit a falsifying interpretation:

$$\forall x(P(x) \odot Q(x)), \exists x P(x) \Rightarrow \exists x Q(x)$$

[5 marks]

- (c) Use the DPLL method to find **all** models (if any) satisfying the following set of formulas.

$$\begin{aligned} (P \wedge R) &\rightarrow Q \\ (\neg Q \wedge R) &\rightarrow P \\ Q &\rightarrow (P \vee R) \\ \neg(P \wedge Q \wedge R) \\ P &\rightarrow R \end{aligned}$$

[5 marks]

- (d) The modal logic S5 differs from S4 in requiring the accessibility relation to be symmetric, as well as reflexive and transitive. Present a formula that is valid for S5 but not for S4. Explain why it is valid for S5 and demonstrate that it is not valid for S4 by drawing an S4 modal frame for which it fails. [5 marks]