## COMPUTER SCIENCE TRIPOS Part II - 2020 - Paper 8

## 1 Advanced Algorithms (tms41)

(a) State the fundamental theorem of Linear Programming.
(b) Consider the following linear program:

$$
\begin{aligned}
& \operatorname{minimise} 4 \cdot x_{1}-x_{2} \\
&-x_{1}+5 x_{2} \geq 4 \\
& x_{1}-0.5 x_{2} \leq 1 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

(i) Convert this linear program into slack form.
(ii) What is the number of different slack forms of the linear program in Part (b) $(i)$ ?
[2 marks]
(iii) Give at least one non-feasible and one feasible basic solution of the linear program in $(b)(i)$.
(c) Consider the following separation problem. We are given $m$ points $x^{1}=$ $\left(x_{1}^{1}, x_{2}^{1}\right), x^{2}=\left(x_{1}^{2}, x_{2}^{2}\right), \ldots, x^{m}=\left(x_{1}^{m}, x_{2}^{m}\right) \in \mathbb{R}^{2}$ and $n$ points $y^{1}=\left(y_{1}^{1}, y_{2}^{1}\right), y^{2}=$ $\left(y_{1}^{2}, y_{2}^{2}\right), \ldots, y^{n}=\left(y_{1}^{n}, y_{2}^{n}\right) \in \mathbb{R}^{2}$. The goal is to find a "separating" vector $w=\left(w_{1}, w_{2}\right) \in \mathbb{R}^{2}$ (if it exists) such that:

$$
\left\langle x^{i}, w\right\rangle=\sum_{j=1}^{2} x_{j}^{i} w_{j}>0 \quad \text { for } i=1,2, \ldots, m,
$$

and

$$
\left\langle y^{i}, w\right\rangle=\sum_{j=1}^{2} y_{j}^{i} w_{j}<0 \quad \text { for } i=1,2, \ldots, n
$$

(i) Create a new, equivalent system of inequalities by replacing each strict inequality by a suitable non-strict inequality. Justify why this new system has a solution if and only if the original system has one.
(ii) Based on your answer in Part $(c)(i)$, how can you solve the above problem using linear programming?
[4 marks]

