COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 8

1 Advanced Algorithms (tms41)

- (a) State the fundamental theorem of Linear Programming. [3 marks]
- (b) Consider the following linear program:

minimise
$$\begin{aligned} 4 \cdot x_1 - x_2 \\ -x_1 + 5x_2 &\geq 4 \\ x_1 - 0.5x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

- (*i*) Convert this linear program into slack form. [3 marks]
- (*ii*) What is the number of different slack forms of the linear program in Part (b)(i)? [2 marks]
- (*iii*) Give at least one non-feasible and one feasible basic solution of the linear program in (b)(i). [4 marks]
- (c) Consider the following separation problem. We are given m points $x^1 = (x_1^1, x_2^1), x^2 = (x_1^2, x_2^2), \ldots, x^m = (x_1^m, x_2^m) \in \mathbb{R}^2$ and n points $y^1 = (y_1^1, y_2^1), y^2 = (y_1^2, y_2^2), \ldots, y^n = (y_1^n, y_2^n) \in \mathbb{R}^2$. The goal is to find a "separating" vector $w = (w_1, w_2) \in \mathbb{R}^2$ (if it exists) such that:

$$\langle x^i, w \rangle = \sum_{j=1}^2 x^i_j w_j > 0$$
 for $i = 1, 2, ..., m$,

and

$$\langle y^i, w \rangle = \sum_{j=1}^2 y^i_j w_j < 0 \quad \text{for } i = 1, 2, ..., n.$$

- (i) Create a new, equivalent system of inequalities by replacing each strict inequality by a suitable non-strict inequality. Justify why this new system has a solution if and only if the original system has one. [4 marks]
- (*ii*) Based on your answer in Part (c)(i), how can you solve the above problem using linear programming? [4 marks]