

COMPUTER SCIENCE TRIPOS Part IB – 2020 – Paper 6

4 Complexity Theory (ad260)

Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a *constructible* function.

(a) Explain why $\text{NTIME}(f) \subseteq \text{SPACE}(f)$ [4 marks]

(b) Explain why any language $L \in \text{NSPACE}(f)$ is also in $\text{TIME}(c^{(f(n)+\log n)})$ for some constant c . [6 marks]

(c) A deterministic machine is a special case of a non-deterministic machine. Explain what this and the results above tell us about the inclusions among the following complexity classes:

$L, NL, P, NP, PSPACE, NPSPACE.$

[5 marks]

(d) Savitch proved that there is an algorithm for the graph reachability problem that uses $O((\log n)^2)$ space. What further inclusions can you derive among the above complexity classes using this fact? Explain your answer. [5 marks]