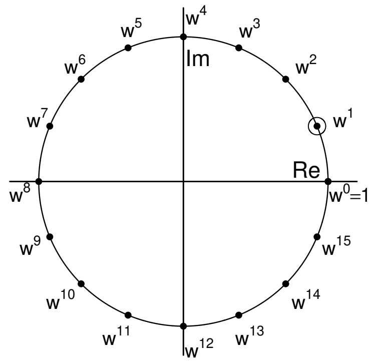
9 Information Theory (jgd1000)

(*a*) If I pick a number n that can be any integer from 1 to ∞ whose probability distribution of being selected is $(\frac{1}{2})^n$, and you ask a series of 'yes/no' questions which I will answer truthfully, how many such 'yes/no' questions should you expect to ask before discovering which number I have picked? Justify your answer by invoking a known series limit. What sequence of questions would be the most efficient to ask, and why? [4 marks]

(*b*) An inner product space containing complex functions $f(x)$ and $g(x)$ is spanned by a set of orthonormal basis functions $\{e_i\}$. Complex coefficients $\{\alpha_i\}$ and $\{\beta_i\}$ therefore exist such that $f(x) = \sum_i \alpha_i e_i(x)$ and $g(x) = \sum_i \beta_i e_i(x)$.

Show that the inner product $\langle f, g \rangle = \sum_i \alpha_i \overline{\beta_i}$. [4 marks]

(*c*) Consider a data sequence $f[n]$ $(n = 0, 1, \ldots, 15)$ having Fourier coefficients $F[k]$ $(k = 0, 1, \ldots, 15)$. Using the 16$^{\text{th}}$ roots of unity labelled around the unit circle as powers of w^1, the primitive 16$^{\text{th}}$ root of unity, construct a sequence of the w^i that could be used to compute $F[3]$ when an inner product is computed between your sequence of w^i and the data sequence $f[n]$.



[4 marks]

(*d*) Explain how vector quantisation exploits sparseness to construct very efficient codes. Use the example of encoding a natural language lexicon with a 15 bit coding budget. Contrast the strategy of using codewords for single letters versus using codewords as pointers to a sparse index of combinations of letters. [4 marks]

(*e*) A continuous signal $f(t)$ has Fourier transform $F(\omega)$. Explain why computing derivatives of $f(t)$ such as $f'(t)$ or $f''(t)$ amounts simply to high-pass filtering. For the n^{th} derivative $f^{(n)}(t)$, what exactly is this filtering operation when expressed in terms of $F(\omega)$? Show how this operation could be used to define derivatives of non-integer order (for example the 1.5$^{\text{th}}$ derivative). [4 marks]