8 Hoare Logic and Model Checking (caw77)

This question is about modelling a program, defined below, consisting of two threads and a single (mathematical) integer variable `X`, initially set to 0. Each thread t has its own program counter given by pc_t, initially set to 0, which describes the *current line* for that thread.

```
Thread 1          Thread 2
0:  X := X+1   ‖  0:  IF IS_ODD(X) THEN STOP_ALL
1:  GOTO 0     ‖  1:  GOTO 0
```

The program is executed by repeatedly carrying out execution steps, where one thread is non-deterministically selected, its entire current line is run, and its program counter is then updated appropriately. This continues until `STOP_ALL` is executed, which immediately terminates the whole program.

(*a*) The program state can be described by $(pc_1, pc_2, \mathtt{X}, \mathit{stopped})$, where pc_1, pc_2, and `X` are mathematical integers, and *stopped* is a boolean which is true iff `STOP_ALL` has been executed. Let S be the set of all such states.

(*i*) Define S_0, the set of initial states of the program, such that $S_0 \subseteq S$. [1 mark]

(*ii*) Define a transition relation $R \subseteq S \times S$ describing the program's execution. [2 marks]

(*iii*) Define a labelling function L that labels all states where the program has terminated with the atomic property `term`. [2 marks]

(*b*) Explain why, taking the definitions from (*a*), the model $M_{\mathbf{a}} = (S, S_0, R, L)$ is *not* a (finite) Kripke structure. [2 marks]

(*c*) Draw the finite state automaton for a model $M_{\mathbf{b}}$ which *is* a Kripke structure, such that $M_{\mathbf{a}}$ and $M_{\mathbf{b}}$ are bisimilar. Justify your answer briefly.
[*Note:* A full formal proof of bisimilarity is not required.] [5 marks]

(*d*) (*i*) Give an LTL formula ϕ such that the judgement $M_{\mathbf{b}} \vDash \phi$ corresponds to the statement "every execution of the program will eventually terminate". [2 marks]

(*ii*) Either prove that $M_{\mathbf{b}} \vDash \phi$ holds, or describe a counter-example trace. [2 marks]

(*e*) Consider the CTL formula $\psi = \mathbf{AG}(\mathbf{EF}\ \mathtt{term})$. Determine whether this is equivalent to your definition of ϕ from Part (*d*). ϕ and ψ are equivalent iff, for all Kripke structures M, $(M \vDash \phi)$ iff $(M \vDash \psi)$. Justify your answer. [4 marks]