COMPUTER SCIENCE TRIPOS Part II – 2019 – Paper 9

14 Types (nk480)

(*a*) In System F, give a Church encoding for (i) the Boolean type, (ii) the definition of the True and False constants, and (iii) the type and definition of the if-then-else operation. [3 marks]

(*b*) In System F, give (i) a Church encoding Nat for the natural numbers, (ii) a Church encoding for the $\mathsf{Zero} : \mathsf{Nat}$ and $\mathsf{Succ} : \mathsf{Nat} \to \mathsf{Nat}$ constructors, and (iii) a type and definition for the iteration operator Iter for natural numbers. [3 marks]

(*c*) (*i*) In System F, give a Church encoding for (i) an Option_A type, (ii) the definitions of the $\mathsf{None} : \mathsf{Option}_A$ and $\mathsf{Some} : A \to \mathsf{Option}_A$ operations, and (iii) the type and definition of the case operation on options.

(*ii*) Assume that $n : B$ and $s : A \to B$, and then

(A) Prove that $\mathsf{Case}\,[B]\,n\,s\,\mathsf{None} = n$

(B) Prove that $\mathsf{Case}\,[B]\,n\,s\,(\mathsf{Some}\,x) = s\,x$

[5 marks]

(*d*) In System F, define a predecessor operation $\mathsf{Pred} : \mathsf{Nat} \to \mathsf{Nat}$, which returns Zero if given Zero as an argument, and return n if given $\mathsf{Succ}\,n$ as an argument. [*Hint*: The option type may be useful in formulating this definition.] [8 marks]

(*e*) In System F, define a subtraction operator $\mathsf{Sub} : \mathsf{Nat} \to \mathsf{Nat} \to \mathsf{Nat}$, which is defined to be *saturating*. That is, $\mathsf{Sub}\,m\,n$ returns the difference if $m \geq n$, and returns 0 otherwise. [1 mark]