COMPUTER SCIENCE TRIPOS Part II – 2019 – Paper 8

1   Advanced Algorithms (tms41)

(a) For each of the following claims, state whether it is true or not and give a brief justification.

   (i)   For any linear program with n variables and m constraints, there are at most $\binom{n+m}{m}$ different basic solutions. [2 marks]

   (ii) The Simplex Algorithm has a worst-case polynomial runtime. [2 marks]

   (iii) In each iteration of the Simplex Algorithm, the value of the objective function changes. [2 marks]

   (iv) The auxiliary linear program in INITIALIZE-SIMPLEX always has a feasible solution. [2 marks]

   (v)   The fundamental theorem of linear programming also holds if linear constraints are allowed to be strict. [2 marks]

   (vi) The set of feasible solutions of any linear program forms a convex set. [2 marks]

(b) For the following linear program, write down the auxiliary linear program used by INITIALIZE-SIMPLEX in slack form: [3 marks]

$$
\begin{aligned}
\text{minimize} \quad & -4x_1 + x_2 \\
\text{subject to} \quad & -4x_1 + 2x_2 \geq -4 \\
& x_1 - 6x_2 \leq -3
\end{aligned}
$$

(c) Recall the algorithm for the unweighted vertex cover problem that is based on rounding the solution of a linear program.

   (i)   What is the approximation ratio of this algorithm? [1 mark]

   (ii) Give an example of a graph and the corresponding linear program for which the gap between the linear program solution and optimal solution is as large as possible. [4 marks]

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