COMPUTER SCIENCE TRIPOS Part IB – 2019 – Paper 6

4 Complexity Theory (ad260)

A Boolean formula ϕ is in *conjunctive normal form* (CNF) if it is the conjunction of clauses, each of which is the disjunction of literals. It is said to be in k-CNF (for $k \in \mathbb{N}$) if each clause has exactly k literals in it.

An assignment $\sigma : V \to \{\text{true, false}\}\ \text{of truth values to the variables is a satisfying assignment for a CNF formula <math>\phi$ if it makes at least one literal in each clause of ϕ true. It is said to be a *not-all-equals* assignment for ϕ if it makes at least one literal in each clause of ϕ true and at least one literal in each clause of ϕ false.

Let CNF-SAT denote the problem of determining, given a formula in CNF, whether it has a satisfying assignment.

Let k-SAT denote the problem of determining, given a formula in k-CNF, whether it has a satisfying assignment.

Let k-NAE denote the problem of determining, given a formula in k-CNF, whether it has a not-all-equals assignment.

- (a) Explain why CNF-SAT is NP-complete. Your explanation should include a full definition of NP-completeness and a brief sketch of the proof of the Cook-Levin theorem.
 [5 marks]
- (b) Show that 3-SAT is NP-complete by means of a suitable reduction. [3 marks]
- (c) Give a polynomial-time reduction from 3-SAT to 4-NAE. What can you conclude about the complexity of the latter problem?
 (*Hint:* consider introducing one new variable and adding it to every clause.)
 [8 marks]
- (d) Show that the problem 3-NAE is NP-complete. (*Hint:* consider a reduction from 4-NAE) [4 marks]