

COMPUTER SCIENCE TRIPOS Part IA – 2019 – Paper 1

1 Foundations of Computer Science (am21)

Three alternative representations for non-negative integers, n , are:

- **Peano**: values have the form $S(\dots S(Z) \dots)$, applying S n times to Z where S and Z are constructors or constants of some data type.
 - **Binary**: values are of type `bool list` with 0 being represented as the empty list, and the least-significant bit being stored in the head of the list.
 - **Church**: values have the form $\text{fn } f \Rightarrow \text{fn } x \Rightarrow f(\dots f(x) \dots)$, applying f n times to x
- (a) Write ML functions for *each* of these data types which take the representation of an integer n as argument and return n as an ML `int`. [6 marks]
- (b) Write ML functions for *each* of these data types which take representations of integers m and n and return the representation of $m + n$. Your answers must *not* use any value or operation on type `int` or `real`. [*Hint*: you might find it useful to write a function `majority: bool*bool*bool -> bool` (which returns true when two or more of its arguments are true) and to note that the ML inequality operator '`<>`' acts as exclusive-or on `bool`.] [10 marks]
- (c) Letting *two* and *three* respectively be the Church representations of integers 2 and 3, indicate whether each of the following ML expressions give a Church representation of some integer and, if so what integer is represented, and if not giving a one-line reason.
- (i) *two three*
- (ii) *three two*
- (iii) *two o three*
- (iv) *three o two*

[4 marks]