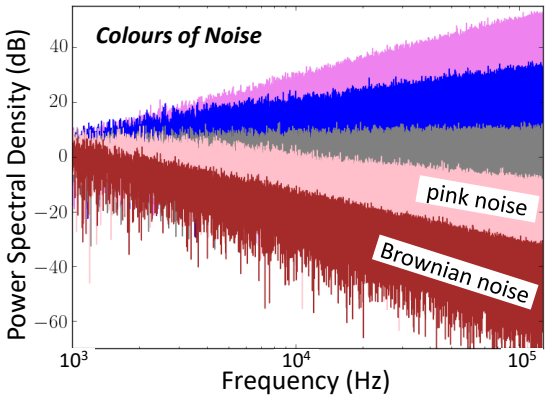
8 Information Theory (JGD)

(*a*) Shannon's *Noisy Channel Coding Theorem* showed how the capacity C of a continuous communication channel is limited by added white Gaussian noise; but other colours of noise are available. Among the "power-law" noise profiles shown in the figure as a function of frequency ω, Brownian noise has power that attenuates as $(\frac{\omega}{\omega_0})^{-2}$, and pink noise as $(\frac{\omega}{\omega_0})^{-1}$, above some minimum ω_0.



Consider three channels suffering from either white, pink, or Brownian noise. At frequency $\omega = \omega_0$ all three channels have the same signal-to-noise ratio SNR(ω_0) and it remains at this level for the white channel, but at higher frequencies ω it improves as $(\frac{\omega}{\omega_0})$ for the pink channel and as $(\frac{\omega}{\omega_0})^2$ for the Brownian channel. Show that across any frequency band $[\omega_1, \omega_2]$ $(\omega_0 < \omega_1 < \omega_2)$ the Brownian and the pink noise channels have higher capacity than the white noise channel, and show that as frequency grows large the Brownian channel capacity approaches twice that of the pink channel. [7 marks]

(*b*) Random variable X can be any of $N = 64$ possible events, about which you have some imperfect knowledge by observing random variable Y. The average uncertainty remaining about X, given observations Y, is $H(X|Y) = 3$ bits. Use Fano's Inequality to estimate a lower bound on the probability of error P_e when guessing X from observations Y. [5 marks]

(*c*) Consider three variable-length codes for a four-symbol alphabet {A,B,C,D} having probabilities $p(x)$ as shown:

x	$p(x)$	Code 1	Code 2	Code 3
A	1/4	00	10	01
B	1/2	1	0	0
C	1/8	01	110	011
D	1/8	10	111	111

Compare the average codeword length of each code to the entropy of the alphabet, and for each code give all possible decodings of the bit sequence '1001' as a complete message. Which codes are uniquely decodable; which have the prefix (instantaneous) property; which code is the best, and why? [8 marks]