5 Digital Signal Processing (MGK)

Your friend Sam works on a physics experiment. This generates a voltage waveform $v(t)$ that is the sum of several signals:

- a sine wave $s(t) = A \cdot \sin(2\pi t f + \phi)$, the frequency f and phase ϕ of which are not known in advance, but f will be within 9.6 kHz $< f <$ 12.0 kHz;
- several other sine waves with frequencies below 8 kHz that Sam needs to ignore in her measurements;
- low levels of noise at all frequencies.

Sam needs to estimate the amplitude A of $s(t)$. She uses a USB audio recorder with a built-in 16 kHz anti-aliasing low-pass filter to digitize $v(t)$ at sampling frequency f_s = 48 kHz, recording s = 100 000 consecutive samples, resulting in real-valued samples $v_0, \ldots, v_{s-1}$. She implemented this algorithm to estimate A:

1: **input** $v_0, \ldots, v_{s-1}$

2: $b := 1000; \quad c := \left\lfloor \frac{s}{b} \right\rfloor$

3: $w_{k,l} := v_{kb+l}$ for all $0 \le k < c$, $0 \le l < b$

4: $x_{k,n} := \sum_{m=0}^{b-1} w_{k,m} \cdot \mathrm{e}^{-2\pi \mathrm{j} \frac{nm}{b}}$ for all $0 \le k < c$, $0 \le n < b$

5: $y_n := \left| \frac{1}{c} \cdot \sum_{k=0}^{c-1} x_{k,n} \right|$ for all $0 \le n < b$

6: $z := \max\{y_{n_1}, \ldots, y_{n_2}\}$ with $n_1 = 200$, $n_2 = 220$

7: **output** z

(*a*) Sam hopes that $A \approx z \cdot \alpha$ for some calibration constant α. She tries to determine α by connecting the USB audio recorder's input to a calibrated laboratory sine-wave generator set to output an amplitude of "60.0 dBµV". What amplitude A in volts will this test signal $A \cdot \sin(\ldots)$ have? [3 marks]

(*b*) When Sam varies the test-signal frequency f in the range 9.6–12.0 kHz, she is disappointed that the output z varies greatly: for some f it even drops to zero!

Describe what Sam's algorithm tries to do, identify and explain *three* problems in it, and change *three* lines to make z more proportional to A across the expected range of f, and close to zero outside that range. [15 marks]

(*c*) Suggest a small adjustment to b to accommodate a faster algorithm for one of the above steps. [2 marks]